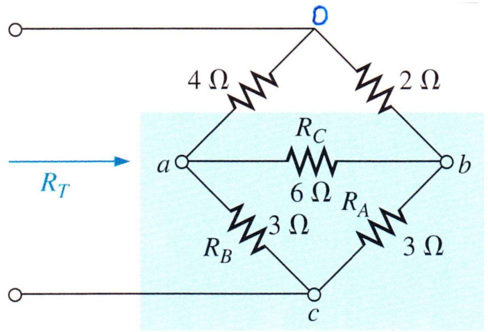
Practice problems for Lesson 4

Practice problems for resistive circuits

B11's Example 8.29

Calculate the equivalent resistance of the circuit shown below.

Let me solve this problem step by step, since it is the first time you see the **er** script.



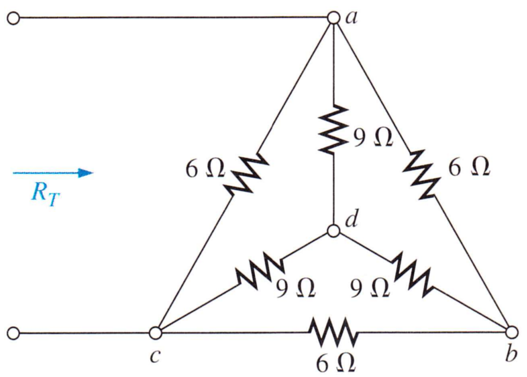
After I label the nodes, I describe the circuit. In this case, I store it in a variable.

"r4,0,a,4:r2,0,b,2:r6,a,b,6:rb,a,c,3:ra,b,c,3"→cir

Run the **er** script, giving it as arguments the circuit and the nodes: s\er(cir,0,c)
When prompted, choose DC as analysis type. Once it is done, evaluate approx(req)
The value is **2.89** Ω. This is correct. Below are many practice examples of this type.

B11's Example 8.30

Find the equivalent resistance of the circuit below.

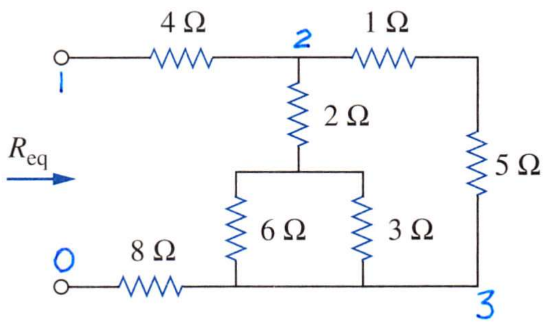


s\er("rac,a,c,6:rad,a,d,9:rab,a,b,6:rcd,c,d,9:rbc,b,c,6:rbd,b,d,9",a,c)

Choose DC. When *Done*, use approx(req) to find the equivalent resistance is **3.27** Ω.

AS2's Example 2.9

Find the equivalent resistance of the circuit below.



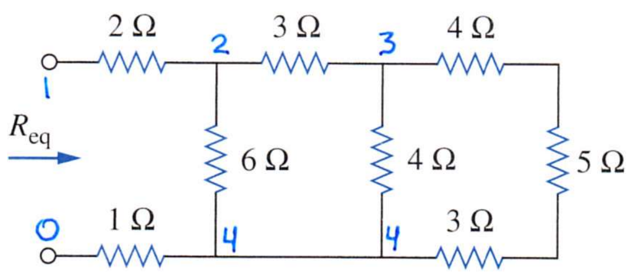
We don't need to run a simulation for this. We can reduce it using s\pr.

4+s\pr({1+5,2+s\pr({6,3})})+8

Evaluating approximately gives us the equivalent resistance: **14.4** Ω.

AS2's Practice Problem 2.9

Find the equivalent resistance of the circuit below.



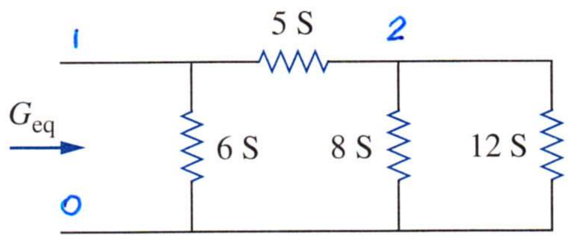
We don't need to run a simulation for this. We can reduce it using s\pr.

2+s\pr({6,3+s\pr({4,4+5+3})})+1

Evaluating approximately gives us the equivalent resistance: **6** Ω.

AS2's Example 2.11

Find the equivalent *conductance* of the circuit below.



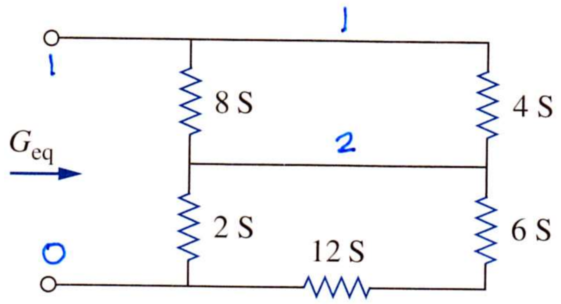
We don't need to run a simulation for this. We can reduce it using s\pr.

1/(s\pr({1/6,1/5+s\pr({1/8,1/12})}))

Evaluating approximately gives us the equivalent conductance: **10** S.

AS2's Practice Problem 2.11

Find the equivalent *conductance* of the circuit below.



We don't need to run a simulation for this. We can reduce it using s\pr.

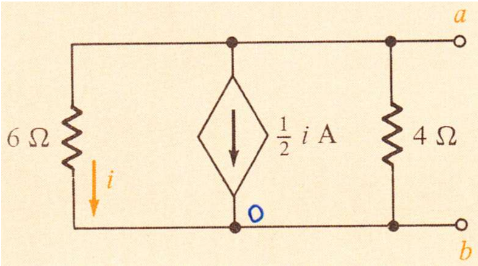
1/(s\pr({1/8,1/4})+s\pr({1/2,1/12+1/6}))

Evaluating approximately gives us the equivalent conductance: **4** S.

Examples with dependent sources

Bo2's Drill Problem 3.14

Find the equivalent resistance of the circuit below.

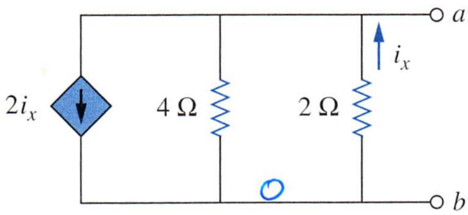


s\er("r4,a,0,4:ri,a,0,6:ji,a,0,iri/2",a,0)

Choose DC. Wait for *Done*. Evaluate req to find the equivalent resistance is **2** Ω.

AS2's Example 4.10

Find the equivalent resistance of the circuit below.

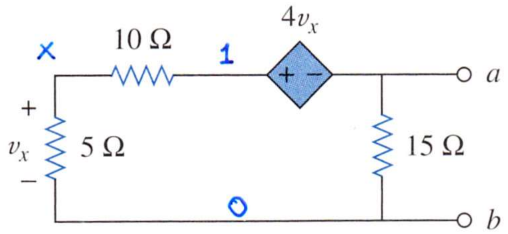


s\er("r4,a,0,4:rx,0,a,2:j,a,0,2irx",a,0)

Choose DC. Wait for *Done*. Evaluate req. The equivalent resistance is **-4** Ω. It may be surprising to have a negative resistance. This is the result of the dependent sources.

AS2's Practice Problem 4.10

Find the equivalent resistance of the circuit below.

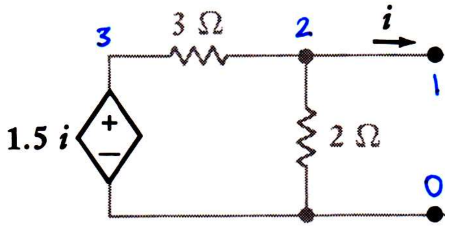


s\er("r15,a,0,15:e,1,a,4vrx:r10,1,x,10:rx,x,0,5",a,0)

Choose DC. Wait for *Done*. Evaluating req approximately, we find the equivalent resistance is **-7.5** Ω.

HK5's Figure 2-29

Find the equivalent resistance of the circuit below.

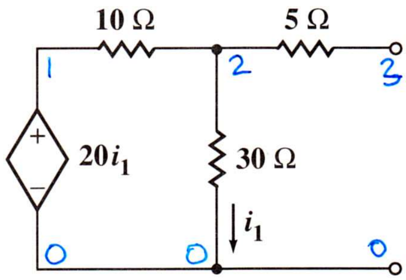


s\er("e,3,0,1.5is:r3,3,2,3:r2,2,0,2:s,2,1",1,0)

Choose DC. Wait for *Done*. Evaluate req. The equivalent resistance is **0.6** Ω.

HK5's Drill Problem 2-9d

Find the equivalent resistance of the circuit below.

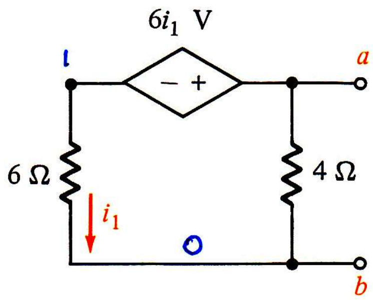


s\er("r10,1,2,10:r5,2,3,5:r1,2,0,30:e,1,0,20ir1",3,0)

Choose DC. Wait for *Done*. Evaluate req. The equivalent resistance is **20** Ω.

Bo2's Example 3.12

Find the equivalent resistance of the circuit below.



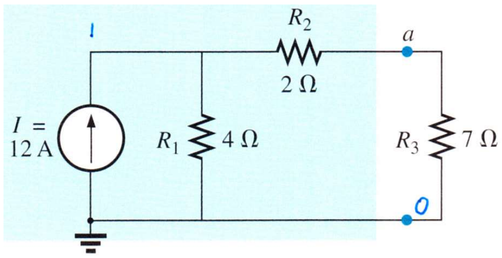
s\er("r1,1,0,6:r4,a,0,4:e,a,1,6ir1",a,0)

Choose DC. Wait for *Done*. Evaluate req. The equivalent resistance is **3** Ω.

Thévenin/Norton problems

B11's Example 9.7

Find the Thévenin equivalent of the circuit below, as seen from the R_3 resistor.

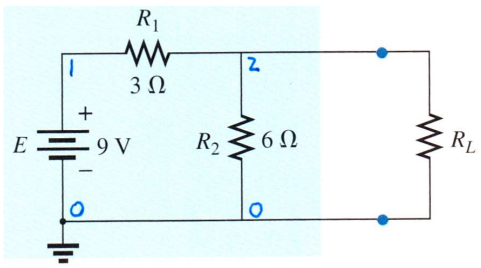


s\th("j,0,1,12:r1,1,0,4:r2,1,a,2",a,0)

Choose DC. Via vth we find V_{TH} = **48** V. Via req we find R_{EQ} = **6** Ω.

B11's Example 9.11

Find the Norton equivalent of the circuit below, as seen from the R_L resistor.

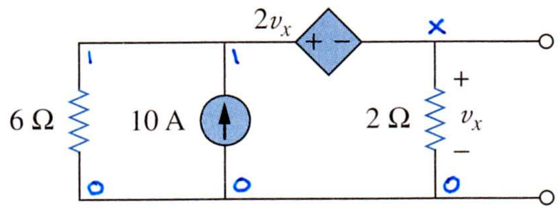


s\th("e,1,0,9:r1,1,2,3:r2,2,0,6",2,0)

Choose DC. Via ino we find I_{NO} = **3** A. Via req we find R_{EQ} = **2** Ω.

AS2's Practice Problem 4.12

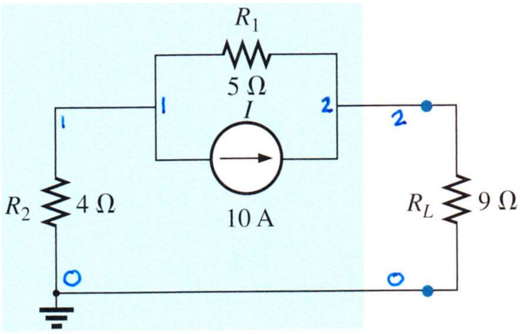
Find the Norton equivalent of the circuit below.



s\th("r6,1,0,6:j,0,1,10:r2,x,0,2:e,1,x,2vx",x,0)

Choose DC. Via ino we find I_{NO} = **10** A. Via req we find R_{EQ} = **1** Ω.

B11's Example 9.12



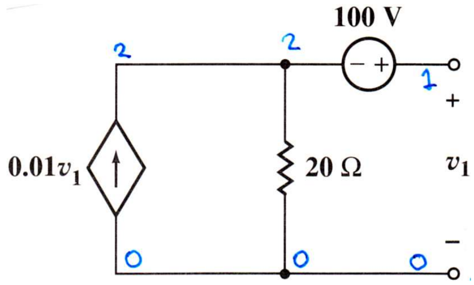
Find the Norton equivalent of the circuit, as seen from the R_L resistor.

s\th("r2,1,0,4:r1,1,2,5:j,1,2,10",2,0)

Choose DC. Via ino we find I_{NO} = **5.56** A. Via req we find R_{EQ} = **9** Ω.

HK5's Drill Problem 2-8b

Find the Thévenin equivalent of the circuit below.

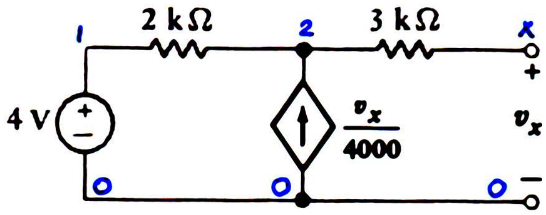


s\th("j,0,2,0.01v1:r,0,2,20:e,1,2,100",1,0)

Choose DC. Via vth we find V_{TH} = **125** V. Via req we find R_{EQ} = **25** Ω.

HK5's Figure 2-27

Find the Thévenin equivalent of the circuit below.



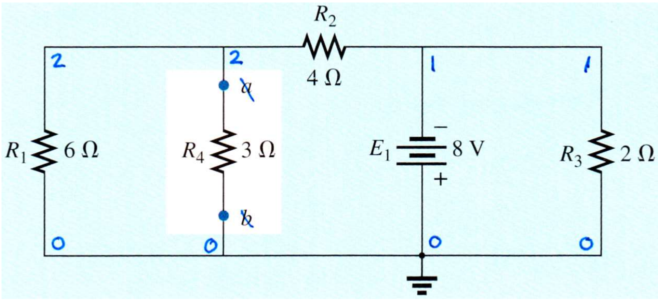
s\th("e,1,0,4:r2,1,2,2'k:r3,2,x,3'k:j,0,2,vx/4000",x,0)

Via vth we find V_{TH} = **8** V. Via req we find R_{EQ} = **10** kΩ.

B11's Example 9.8

Find the Thévenin equivalent of the circuit below, as seen from the R_4 resistor.

I ignore the textbook's decision to call the nodes **a** and **b**, since b is ground anyway.

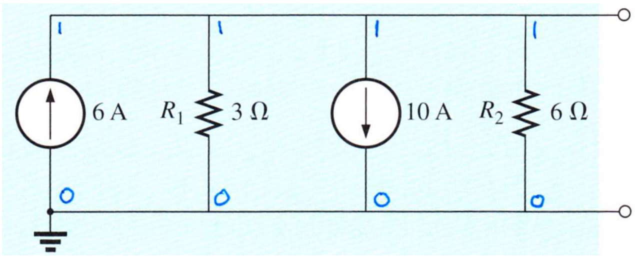


s\th("r1,2,0,6:r2,2,1,4:r3,1,0,2:e,0,1,8",2,0)

Choose DC. Via vth we get V_{TH}= **-4.8** V. Via req we get R_{EQ}= **2.4** Ω.

B11's Example 8.6

Find the Norton equivalent of the circuit below.

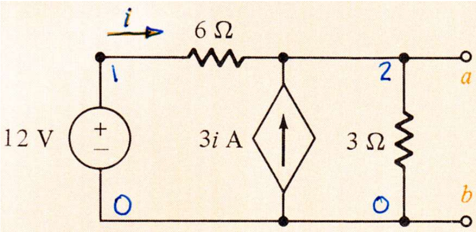


s\th("j1,0,1,6:r1,1,0,3:j2,1,0,10:r2,1,0,6",1,0)

Choose DC. Via ino we find I_{NO} = **-4** A. Via req we find R_{EQ} = **2** Ω.

Bo2's Drill Exercise 3.12

Find the Norton equivalent of the circuit below.

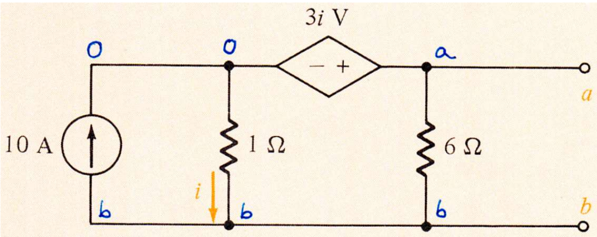


s\th("e,1,0,12:r6,1,2,6:j,0,2,3ir6:r3,2,0,3",2,0)

Choose DC. Via ino we find I_{NO} = **8** A. Via req we find R_{EQ} = **1** Ω.

Bo2's Drill Exercise 3.9

Find the Thévenin equivalent of the circuit below.

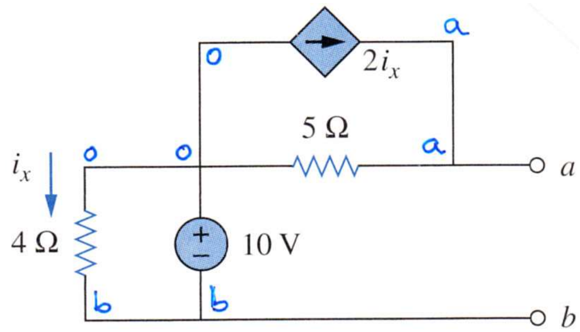


s\th("j,b,0,10:r1,0,b,1:e,a,0,3ir1:r6,a,b,6",a,b)

Choose DC. Via vth we find V_{TH} = **24** V. Via req we find R_{EQ} = **2.4** Ω.

AS2's Example 4.12

Find Norton equivalent of the circuit below.

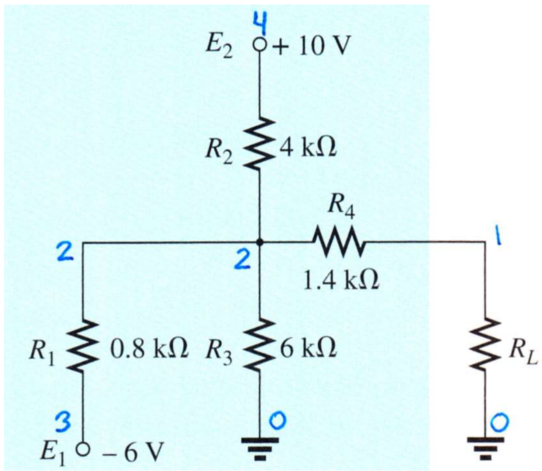


s\th("rx,0,b,4:e,0,b,10:r5,0,a,5:j,0,a,2irx",a,b)

Choose DC. Via ino we find I_{NO} = **7** A. Via req we find R_{EQ} = **5** Ω.

B11's Example 9.10 (Hidden source)

Find the Thévenin equivalent of the circuit below.



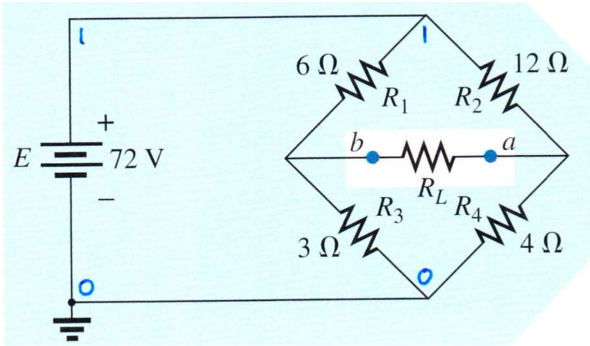
My solution is below:

s\th("e1,3,0,-6:e2,4,0,10:r1,2,3,0.8'k:
r2,2,4,4'k:r3,2,0,6'k:r4,2,1,1.4'k",1,0)

Choose DC. Via vth we find V_{TH} = -**3** V. Via req we find R_{EQ} = **2** kΩ.

B11's Example 9.9

Find the Thévenin equivalent of the circuit below, as seen from the R_L resistor.

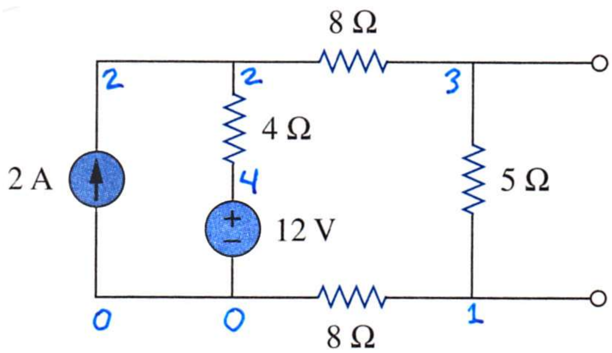


s\th("e,1,0,72:r1,1,b,6:r2,1,a,12:r3,0,b,3:r4,0,a,4",b,a)

Choose DC. Via vth we find V_{TH} = **6** V. Via req we find R_{EQ} = **5** Ω.

AS2's Example 4.11

Find the Norton equivalent of the circuit below.

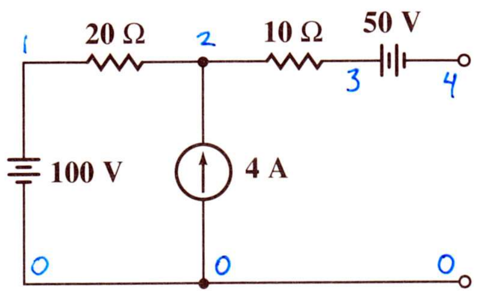


s\th("j,0,2,2:e,4,0,12:r1,2,4,4:r2,2,3,8:r3,0,1,8:r4,3,1,5",3,1)

Choose DC. Via ino we find I_{NO} = **1** A. Via req we find R_{EQ} = **4** Ω.

HK5's Drill Problem 2-8a

Find the Thévenin equivalent of the circuit below.

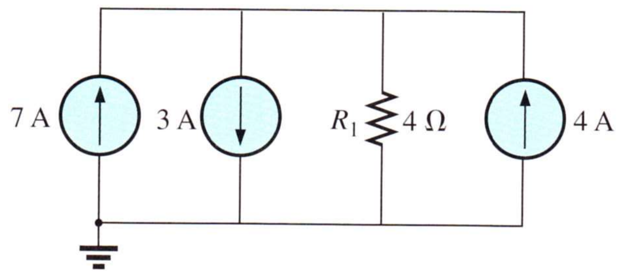


s\th("e1,1,0,100:r2,1,2,20:j,0,2,4:r1,2,3,10:e5,3,4,50",4,0)

Choose DC. Via vth we find V_{TH} = **130** V. Via req we find R_{EQ} = **30** Ω.

B11's Example 8.7

Find Norton equivalent of the circuit below.

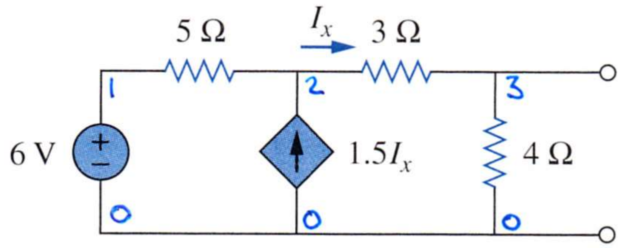


s\th("j7,0,1,7:j3,1,0,3:r1,1,0,4:j4,0,1,4",1,0)

Choose DC. Via ino we find I_{NO} = **8** A. Via req we find R_{EQ} = **4** Ω.

AS2's Practice Problem 4.9

Find the Thévenin equivalent of the circuit below.

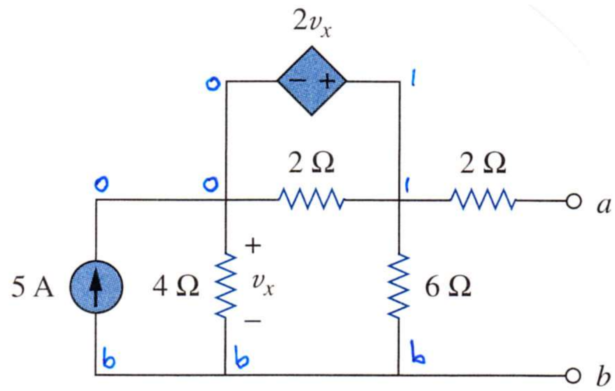


s\th("e,1,0,6:r5,1,2,5:rx,2,3,3:j,0,2,1.5irx:r4,3,0,4",3,0)

Choose DC. Via vth we find V_{TH} = **5.33** V. Via req we find R_{EQ} = **0.44** Ω.

AS2's Example 4.9

Find the Thévenin equivalent of the circuit below.

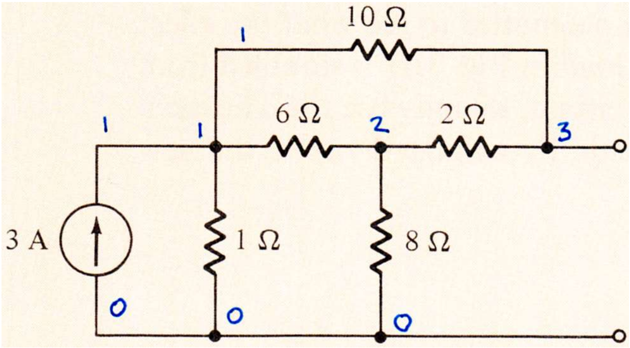


s\th("j,b,0,5:rx,0,b,4:r1,0,1,2:r2,1,b,6:r3,1,a,2:e,1,0,2vrx",a,b)

Choose DC. Via vth we find V_{TH} = **20** V. Via req we find R_{EQ} = **6** Ω.

Bo2's Drill Exercise 3.8

Find the Thévenin equivalent of the circuit below.

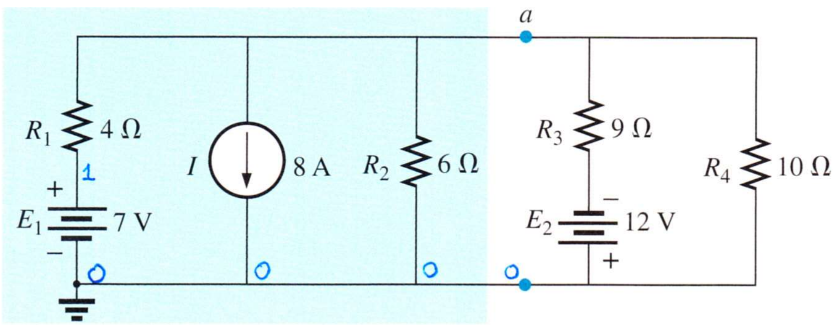


s\th("j,0,1,3:r1,0,1,1:r6,1,2,6:r10,1,3,10:r8,2,0,8:r2,2,3,2",3,0)

Choose DC. Via vth we find V_{TH} = **2** V. Via req we find R_{EQ} = **4** Ω.

B11's Example 9.13

Find Norton equivalent of the shaded part of the circuit.



s\th("e1,1,0,7:r1,1,a,4:j,a,0,8:r2,a,0,6",a,0)

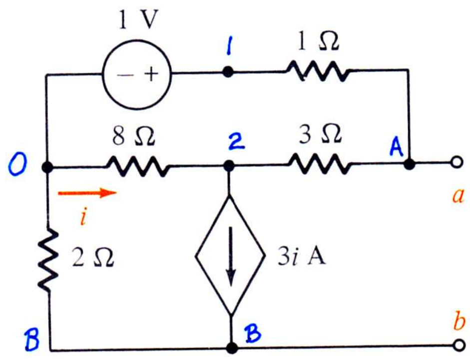
Choose DC. Via ino we find I_{NO} = **-6.25** A. Via req we find R_{EQ} = **2.4** Ω.

Tricky problems

Some circuit theory books – and some professors – find it entertaining or instructive to surprise unsuspecting students with tricky problems, like the two we solve below.

Bo2's Example 3.11 (Tricky)

Find the Norton equivalent of the circuit below.



Since you do not know beforehand that this is a tricky problem, you go for the usual:

```
s\th("r2,0,b,2:r8,0,2,8:r3,2,a,3:r1,1,a,1:e,1,0,1:j,2,b,3ir8",a,b)
```

You choose DC, press Enter and wait. Symbulator reports the calculator was unable to solve the equations. This often means there is a division by zero somewhere.

Clean the variables from the MAIN folder and try again, this time – following Symbulator's advise –using a symbolic value. We chose to use **x** instead of **3** in the dependent source.

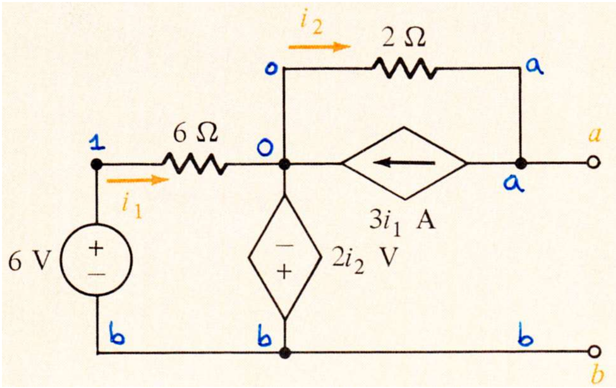
```
s\th("r2,0,b,2:r8,0,2,8:r3,2,a,3:r1,1,a,1:e,1,0,1:j,2,b,x*ir8",a,b)
```

Now the calculator can solve just fine. Evaluate the answers: the expression for `ino` is fine, but the expression for `req`, **(9*x-35)/(4*(x-3))**, will result in a division by zero.

Via `Define x=3: {ino,req}` we find that I_{NO} = **1** A, and R_{EQ} is undefined. This means the equivalent resistance is, for practical purposes, infinite. Your idea of fun, right?

Bo2's Drill Exercise 3.13 (Tricky)

Below is another tricky problem. Find the Norton equivalent of the circuit below.



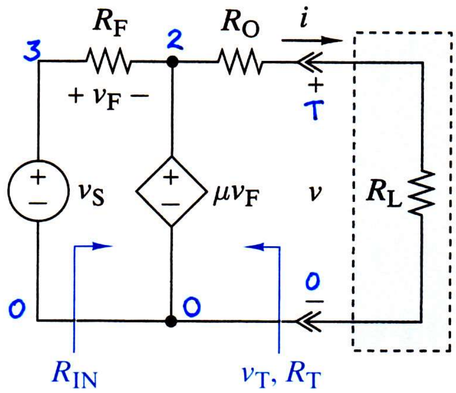
As in the previous example, we get an error if we simulate using **2** and **3** in the dependent sources. So, we use **2x** and **3x** instead, where **x** will be later defined as 1.

```
s\th("ei,1,b,6:ed,b,0,2*x*ir2:r1,1,0,6:r2,0,a,2:jd,a,0,3*x*ir1",a,b)
```

Exploring the answers, we see that the denominator of the expression for `req`, (x-1), given that x=1), results in a division by zero. Via `1→x:{ino,req}` we find that I_{NO} = -3A, and R_{EQ} is undefined or, for practical purposes, infinite:

TR5's Exercise 4-6 (Symbolic)

Find the input resistance and output Thévenin equivalent circuit of the circuit



$$R_{IN} = (1 + \mu)R_F$$

$$v_T = \frac{\mu}{\mu + 1} v_S$$

$$R_T = R_O$$

First let's find the input resistance, R_{IN}, e.g. the resistance as seen by the v_S source. We use **μ** for the constant in the dependent source.

```
s\dc("ei,3,0,vs:rf,3,2,rf:ro,2,t,ro:ed,2,0,μ*vrf"):rei
```

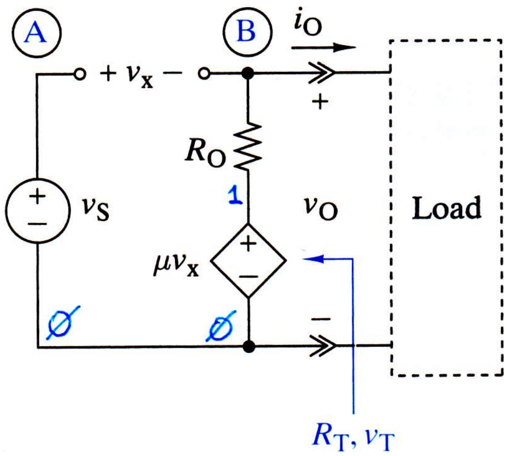
We get rf*(μ+1), which is correct. The textbook's answers are shown right of the circuit schematic. Now we find the output Thévenin equivalent circuit as seen by R_L.

```
s\th("ei,3,0,vs:rf,3,2,rf:ro,2,t,ro:ed,2,0,μ*vrf",t,0):{vth,req}
```

We get **{vs*μ/(μ+1),ro}**, which is correct, as can be seen in the textbook's answers for v_T and R_T, shown right of the circuit schematic above.

TR5's Example 4-8 (Symbolic)

Find the Thévenin equivalent as seen by the load.



My answer starts by defining ***v_x*** as ***va-vb***. This is done thus:

```
Define vx=va-vb
```

Now I run the th script, with the circuit description shown below:

```
s\th("ei,a,0,vs:ed,1,0,μ*(vx):ro,b,1,ro",b,0):
{vth,req}
```

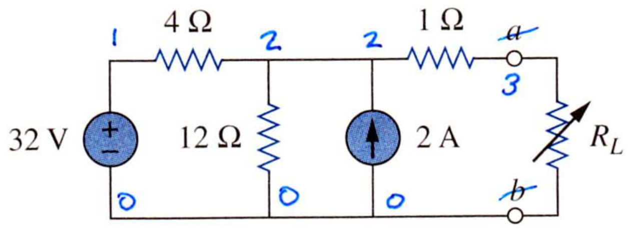
The answers we get, **{vs*μ/(μ+1),ro/(μ+1)}**, are correct, as can be seen by comparing them to those in the book:

$$v_T = \frac{\mu v_S}{\mu + 1} \quad \text{and} \quad R_T = \frac{R_O}{\mu + 1}$$

I am not sure there is any other circuit simulator for calculators that can do this.

AS2's Example 4.8

Find the Thévenin equivalent of the circuit shown to the left of terminals a-b. Then find the current through RL = 6, 16 and 36Ω.



My solution below.

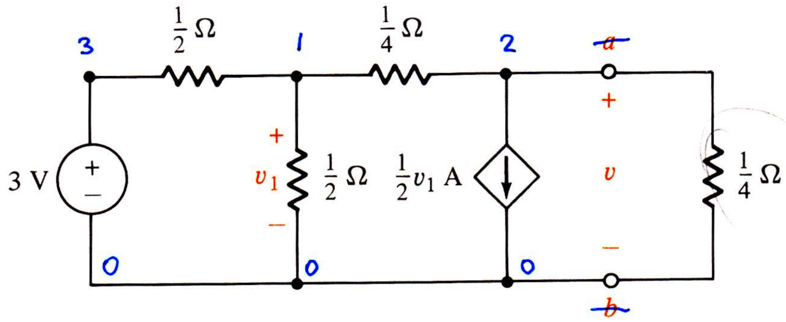
```
s\th("e,1,0,32.:r4,1,2,4:r12,2,0,12:j,0,2,2:r1,2,3,1",3,0):
{vth,req,irL|Load=6,irL|Load=16,irL|Load=36}
```

Select DC when asked. Answer Y when offered the load formulas.

The answer, **{30.,4.,3.,1.5,.75}**, is correct.

Bo2's Example 3.10

Find the Norton equivalent of the circuit left of the a-b terminals, and then find the voltage drop and the current through the ¼ Ω resistor. My one-line solution below.



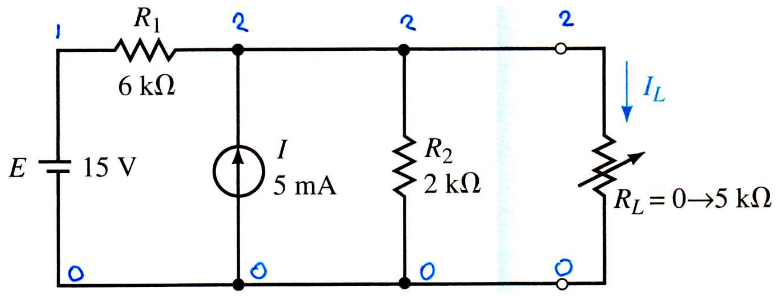

```
s\th("e,3,0,3:r31,3,1,1/2:r10,1,0,1/2:r12,1,2,1/4:j,2,0,v1/2",2,0):
{ino,req,vrL|Load=1/4,irL|Load=1/4}
```

Select DC when asked. Answer Y when offered the load formulas.

The book gives the answers as fractions. We get it right: **{21/8,4/9,21/50,42/25}.**

RM3's Example 9-7

Find the Norton equivalent of the circuit external to R_L. Then determine the load current I_L when R_L = 0 Ω, 2'k Ω and 5'k Ω. My one-line solution below.



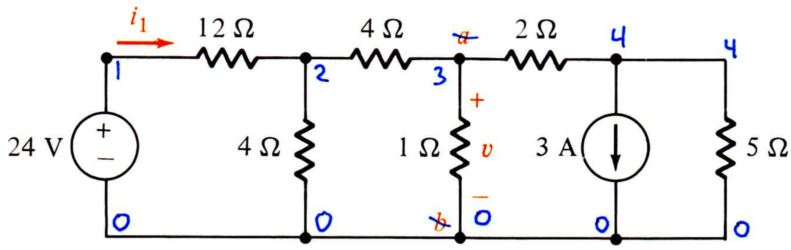

```
s\th("e,1,0,15.:r1,1,2,6'k:j,0,2,5'm:r2,2,0,2'k",2,0):
{ino,req, irL|Load=0,irL|Load=2000,irL|Load=5000}
```

Select DC when asked. Answer Y when offered the load formulas.

The answer, **{.0075,1500.,.0075,.00321,.00173}**, is correct.

Bo2's Example 3.5

Find the Norton equivalent of the circuit external to the 1Ω resistor. Then determine the voltage drop across this 1Ω resistor.



My solution below.

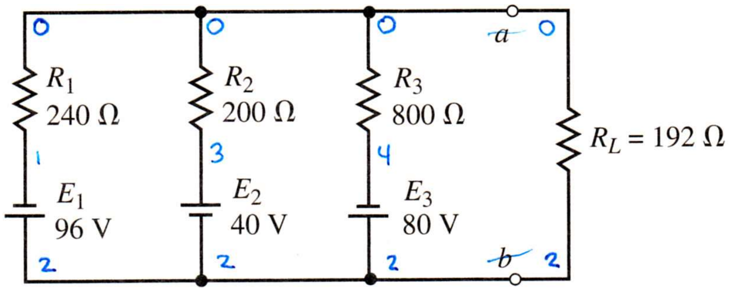
```
s\th("e,1,0,24:r12,1,2,12:r20,2,0,4:r23,2,3,4:r34,3,4,2:
j,4,0,3:r40,4,0,5",3,0):{ino,req,vrL|Load=1}
```

Select DC when asked. Answer Y when offered the load formulas.

The answer, **{-9/7,7/2,-1}**, is correct.

RM3's Example 9-13

Use Millman's Theorem to simplify the circuit left of a-b so that there is only one voltage and one resistor. Then find the current in the load resistor R_L.



I don't know Millman's Theorem, but in my book this is called the Thévenin equivalent.

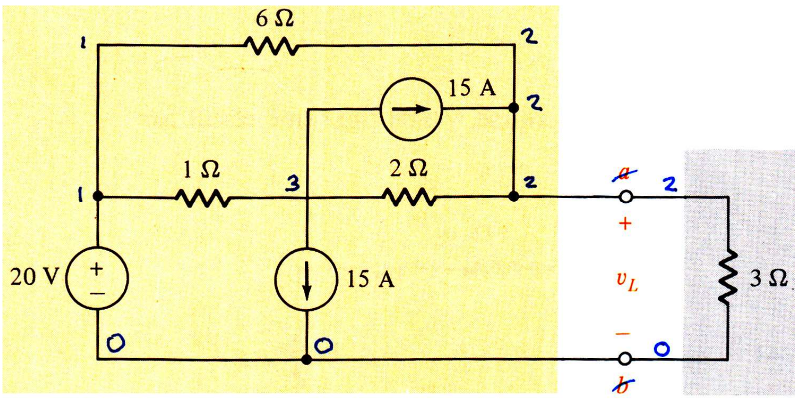
```
s\th("r1,0,1,240.:e1,2,1,96:r2,0,3,200:e2,3,2,40:
r3,0,4,800:e3,2,4,80",2,0):{vth,req,irL|Load=192}
```

Select DC when asked. Answer Y when offered the load formulas.

The answer, **{28.8,96.,.1}**, is correct.

Bo2's Example 3.7

Find the Thévenin equivalent for the circuit left of a-b. Then find the voltage across the 3 Ω resistor, and also if it was 6 Ω. My answer is presented below.




```
s\th("e,1,0,20:r6,1,2,6:r1,1,3,1:r2,3,2,2:j32,3,2,15:j30,3,0,15",2,0):
{vth,req,vrL|Load=3,vrL|Load=6.}
```

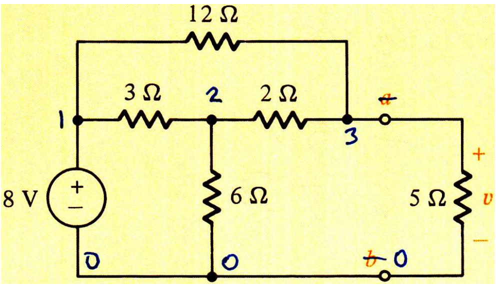
Select DC when asked. Answer Y when offered the load formulas.

The answer we find, **{30,2,18,22.5}**, is correct.

Bo2's Drill Exercise 3.7

Find the Thévenin equivalent for the circuit left of a-b. Then find the voltage *v*.

My answer below.



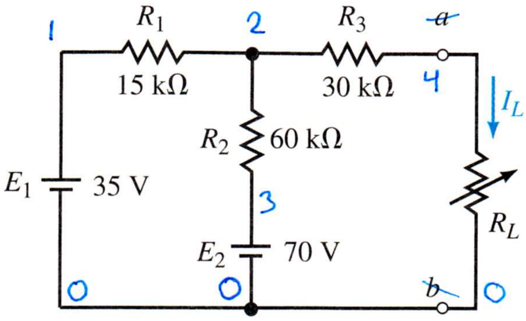
s\th("e,1,0,8:r3,1,2,3:r12,1,3,12:r6,2,0,6:r2,2,3,2",3,0):
{vth,req,vrL|Load=5.}

Select DC when asked. Answer Y when offered the load formulas.

The answer, **{6,3,3.75},** is correct.

RM3's Practice Problem 9.5

Find the Norton equivalent external to R_L in the circuit. Solve for the current I_L when $R_L = 0$, 10 kΩ, 50 kΩ, and 100 kΩ.



Answers
$R_N = 42$ kΩ, $I_N = 1.00$ mA
For $R_L = 0$: $I_L = 1.00$ mA
For $R_L = 10$ kΩ: $I_L = 0.808$ mA
For $R_L = 50$ kΩ: $I_L = 0.457$ mA
For $R_L = 100$ kΩ: $I_L = 0.296$ mA

My solution below.

s\th("e1,1,0,35.:r1,1,2,15'k:r2,2,3,60'k:e2,3,0,70:r3,2,4,30'k",4,0):
{ino,req,irL|Load=0,irL|Load=1E4,irL|Load=5E4,irL|Load=1E5}

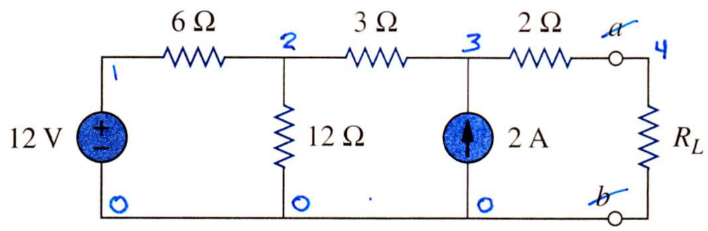
Select DC when asked. Answer Y when offered the load formulas.

The answer, **{.001,42000,.001,.000808,.000457,.00296}**, is correct.

Power transfer problems

AS2's Example 4.13

Find the R_L value for maximum power transfer and the maximum power transferred.



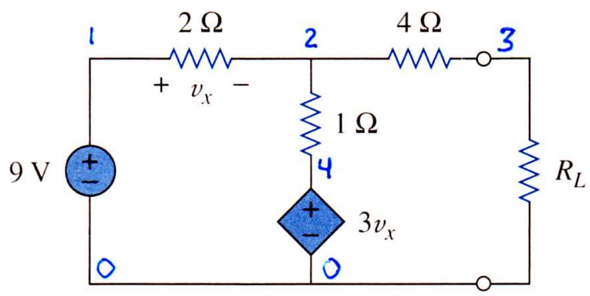
R_L for maximum transfer is **rEq**. The maximum power is in **pMax**. My solution below:

```
s\th("e,1,0,12:r6,1,2,6:r12,2,0,12:r3,2,3,3:j,0,3,2:r2,3,4,2",4,0):
approx({req,pmax})
```

Select DC. You can answer N when asked about the load equations. The answer, **{9.,13.44}**, is correct: the maximum transfer of power occurs when the load is 9Ω. At this point, the power transferred is 13.44W. Now let's solve another one.

AS2's Practice Problem 4.13

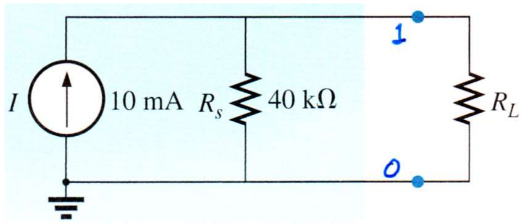
Find the R_L value for maximum power transfer and the maximum power transferred.




```
s\th("ei,1,0,9:rx,1,2,2:r1,2,4,1:ed,4,0,3vrx:r4,2,3,4",3,0):
approx({req,pmax})
```

Select DC. You can answer N. The logic of this problem is identical to the previous one. The answer is **{4.22,2.901}.**

B11's Example 9.15



a. Find the load resistance that will result in maximum power transfer to the load, and find the maximum power delivered.
b. If the load were changed to 68 kΩ, would you expect a fairly high level of power transfer to the load based on the results of part (a)? What would the new power level be? Is your initial assumption verified?
c. If the load were changed to 8.2 kΩ, would you expect a fairly high level of power transfer to the load based on the results of part (a)? What would the new power level be? Is your initial assumption verified?

My one-line solution to all three questions is presented below.

```
s\th("j,0,1,10'm:rs,1,0,40'k",1,0):
{req,pmax,prl|load=68000.,prl|load=8200.}
```

Select DC and answer Y about the load formulas. The answer we obtain, **{40000,1,.93,.57}**, is correct. Let's deconstruct it.

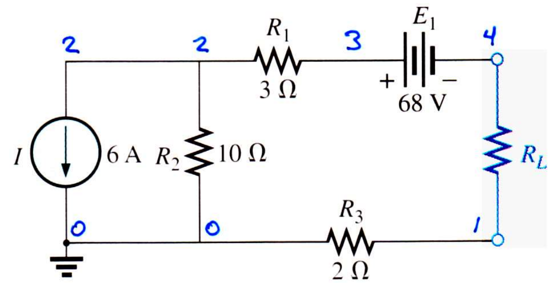
Part (a) is answered by the first two values: a 40'k Ω resistor as load would receive 1W power. Since this is the maximum – this is the most that any load could receive ever.

Parts (b) is answered by the third value. Making use of the variable **prL**, which contains the power delivered by the circuit equivalent to the load, as a function of the load value **L**, we find that a load of 68'kΩ receives .93W, which is less than the maximum.

Parts (c) is answered in similar manner by the fourth value. A load of 8.2'kΩ receives .57W, which is less than the maximum. Any resistance other than 40'k gets less power.

B11's Example 9.17

Find the R_L value for maximum power transfer and the maximum power transferred.




```
s\th("j,2,0,6:r2,2,0,10:r1,2,3,3:r3,0,1,2:e,3,4,68",4,1):
approx({req,pmax})
```

Select DC. You can answer N. Answer is **{15.,273.07}.** Let's now see one that is a little different.