# Practice Problems for Lesson 6

## Transient analysis of RC circuit

### Bo2's Example 5.1

EXAMPLE 5.1

Let us begin this example by considering the circuit in Fig. 5.4, in which there is a switch that is closed for time $t < 0$, is opened at time $t = 0$, and stays open for all time $t > 0$. For this circuit, let us determine $v_C(t)$ for all values of $t$.

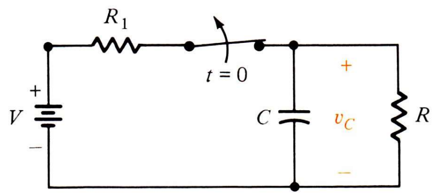



Fig. 5.4 *RC* circuit with a switch.

**Solution.** Here we have two intervals. For the first interval, when **t < 0s**, we assume that things have been steady for a long time now and any transient effect has long passed. So for this first interval of t < 0s, we do a DC simulation to find the initial conditions for the second interval.

```
s\dc("e,1,0,V:r1,1,2,r1:c,2,0,c:r,2,0,r")
```

When the simulation is *Done*, we ask for `vc`, the voltage in the capacitor **c**. We get the expression shown below, which is correct.

$$\frac{r\,v}{r + r1}$$

For the second interval, when **0s ≤ t**, we do a transient simulation. We give the capacitor as initial condition its voltage from the previous interval.

```
s\tr("c,2,0,c,r*v/(r+r1):r,2,0,r”)
```

When the simulation is *Done*, we ask for `vc` again. We get the expression below, which is correct.

$$e^{-t/cr}\,\frac{r\,v}{r + r1}$$

## Bo2's Drill Exercise 5.1

DRILL EXERCISE 5.1

For the circuit shown in Fig. DE5.1, find a single expression for all $t$ for (a) $v_C(t)$ and (b) $i_C(t)$.

Answer: (a) $6 - 6(1 - e^{-4t})u(t)$ V; (b) $-2e^{-4t}$ A

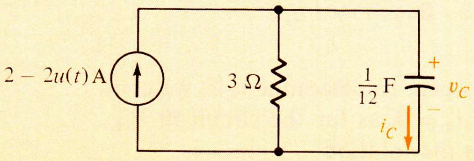



Fig. DE5.1

**Solution.** Again, we have two intervals: the first for DC, the second for TR.

For the first interval, when **t < 0s**, the current source is 2A. Simulate in DC.

s\dc("j,0,1,2:r,1,0,3:c,1,0,1/12"):{vc,ic}

**{ 6 , 0 }**

So the voltage in the capacitor is 6V. We store this value in a variable called **vc$_0$**, to use it as initial condition of the capacitor for the second interval.

vc→vc0

For the second interval, when **0s ≤ t**, we do a transient simulation. The current source is 0A, and the capacitor has an initial condition: voltage **vc$_0$**.

s\tr("j,0,1,0:r,1,0,3:c,1,0,1/12,vc0”):{vc,ic}

When the simulation is *Done*, we get the expressions below. They are right.

**{ $6e^{-4t}$ , $-2e^{-4t}$ }**

## Bo2's p224 5.2

EXAMPLE 5.2

For the circuit shown in Fig. 5.7, let us determine $v_C(t)$, $i_C(t)$, and $v(t)$ for all time.

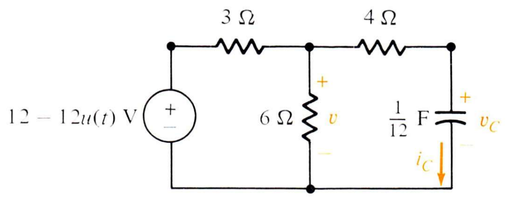



Fig. 5.7 Circuit for Example 5.2.

**Solution.** Again, we have two intervals: the first for DC, the second for TR.

For the interval when **t < 0s**, the voltage source is 12A. Simulate in DC.

```
s\dc("e,1,0,12:r3,1,2,3:r6,2,0,6:r4,2,3,4:c,3,0,1/12"):{vc,ic,v2}
```

$$\{ 8 , 0 , 8 \}$$

For the interval when **0s ≤ t**, the voltage source is 0V. Simulate in TR, giving the capacitor as initial condition its voltage from the previous interval.

```
s\tr("e,1,0,0:r3,1,2,3:r6,2,0,6:r4,2,3,4:c,3,0,1/12,8”):{vc,ic,v2}
```

$$\{ 8e^{-2t} , \frac{-4}{3}e^{-2t} , \frac{8}{3}e^{-2t} \}$$

## Bo2's Drill Exercise 5.2

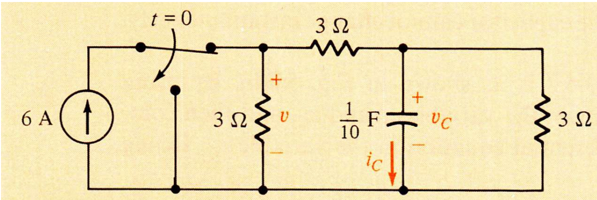



Fig. DE5.2

DRILL EXERCISE 5.2

For the circuit shown in Fig. DE5.2, find an expression for all $t$ for (a) $v_C(t)$, (b) $i_C(t)$, and (c) $v(t)$.

Answer: (a) $6 - 6(1 - e^{-5t})u(t)$ V; (b) $-3e^{-5t}u(t)$ A; (c) $12 - 12u(t) + 3e^{-5t}u(t)$ V

**Solution.** For **t < 0s**, the 6A source is part of the circuit. Simulate in DC.

```
s\dc("j,0,1,6:r10,1,0,3:r12,1,2,3:r20,2,0,3:c,2,0,1/10"):{vc,ic,v1}
```

**{ 6 , 0 , 12 }**

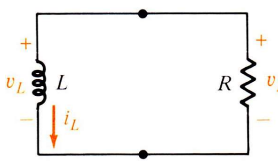
For **0s ≤ t**, the source is no longer relevant and can be excluded from the circuit. Simulate in TR, giving the capacitor its proper initial condition.

```
s\tr("r10,1,0,3:r12,1,2,3:r20,2,0,3:c,2,0,1/10,6"):{vc,ic,v1}
```

$$\{\, 6e^{-5t} \,,\, -3e^{-5t} \,,\, 3e^{-5t} \,\}$$

## Bo2's Figure 5.10a

Look at the simple resistor-inductor (RL) circuit shown below, where at time t=0 the inductor current is $i_L(0)$. Determine $v_L(t)$, $i_L(t)$ and $v_R(t)$ for t≥0.




Since we are already given the initial condition, we only run the transient simulation for t≥0. In our circuit description below, the initial condition is **il0**.

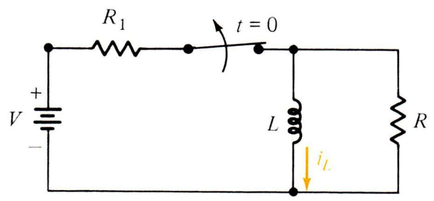
```
s\tr("l,1,0,l,il0:r,1,0,r"):{vl,il,vr}
```

After 22 seconds, we get the right answers:

$$\{\, -il0\; r\; e^{\frac{-r\,t}{l}} \,,\, il0\; e^{\frac{-r\,t}{l}} \,,\, -il0\; r\; e^{\frac{-r\,t}{l}} \,\}$$

## Bo2's Example 5.3

Determine $i_L(t)$ for all t.




**Solution.** For **t < 0s**, simulate in DC.

```
s\dc("e,1,0,v:r1,1,2,r1:l,2,0,l:r2,2,0,r"):il
```

$$\frac{v}{r1}$$

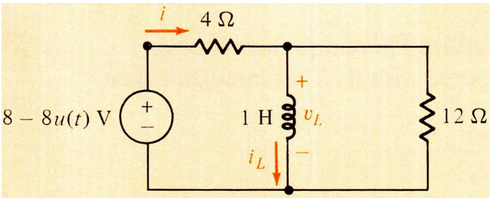
For **t ≥ 0s**, simulate in TR, giving the inductor its initial condition.

```
s\tr("l,2,0,l,v/r1:r2,2,0,r"):il
```

$$e^{\frac{-r\,t}{l}}\,\frac{v}{r1}$$

## Bo2's Drill Exercise 5.3

Find $i_L(t)$, $v_L(t)$ and i(t) for all t.




**Solution.** For **t < 0s**, simulate in DC.

```
s\dc("e,1,0,8:r4,1,2,4:l,2,0,1:r12,2,0,12"):{il,vl,ir1}
```

$$\{\,2\,,\,0\,,\,2\,\}$$

For **t ≥ 0s**, simulate in TR, giving the inductor its initial condition of 2A.

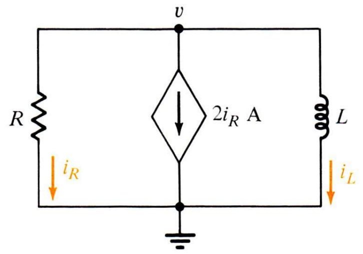
```
s\tr("e,1,0,0:r4,1,2,4:l,2,0,1,2:r12,2,0,12"):{il,vl,ir4}
```

$$\{\,2e^{-3t}\,,\,-6\,e^{-3t}\,,\,\frac{3}{2}e^{-3t}\,\}$$

In my machine the simulation took 30 seconds, 12 seconds of these (40%) were used in finding the inverse Laplace of the answers. Later we will learn a trick to save time by specifying to Symbulator which answers are required.

## Bo2's p230 (Dependent source)

Find $i_L(t)$ for $t \geq 0$, given that $i_L(0) = 5A$.



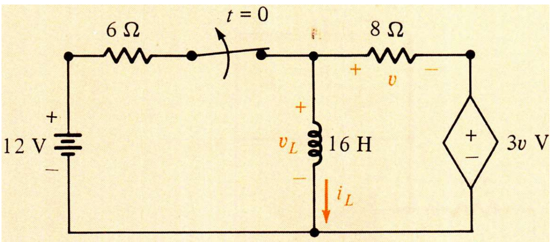

We only run the transient simulation for t≥0, with initial condition 5A.

s\tr("r,v,0,r:j,v,0,2ir:l,v,0,l,5"):il

$$5e^{\left(\frac{-R\,t}{3L}\right)}$$

## Bo2's Drill Exercise 5.4 (Dependent source)

Find $i_L(t)$ and $v_L(t)$ for all t.




**Solution.** For **t < 0s**, simulate in DC. This gives us the initial condition.

s\dc("ei,1,0,12:r6,1,2,6:l,2,0,16:r8,2,3,8:ed,3,0,3vr8"):{il,vl}

$$\{\,2\,,\,0\,\}$$

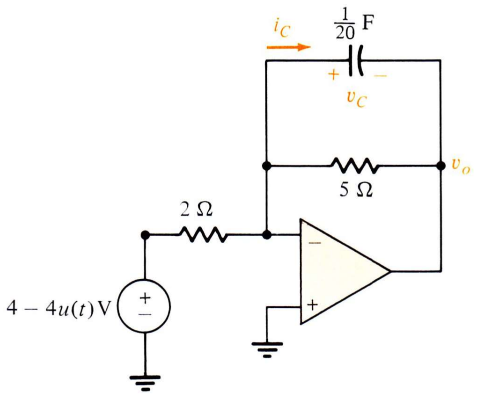
For **t ≥ 0s**, simulate in TR, giving the inductor its initial condition.

s\tr("l,2,0,16,2:r8,2,3,8:ed,3,0,3vr8"):{il,vl}

$$\{\,2\,e^{-2t}\,,\,-64\,e^{-2t}\,\}$$

## Bo2's Example 5.5 (Op Amp)

Determine $v_C(t)$, $i_C(t)$ and $v_o(t)$ for all t.




**Solution.** For **t < 0s**, simulate in DC. This gives us the initial condition.

```
s\dc("e,3,0,4:r2,3,1,2:r5,1,o,5:c,1,o,1/20:o,0,1,o"):{vc,ic,vo}
```

**{ 10 , 0 , -10 }**

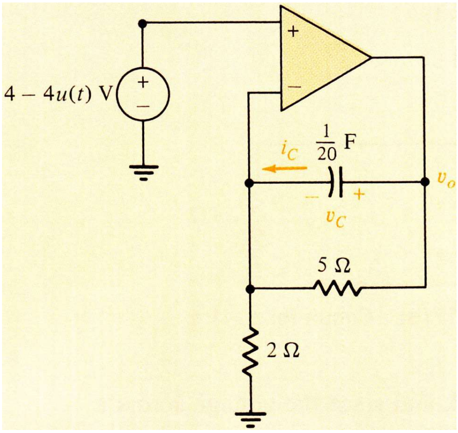
For **t ≥ 0s**, simulate in TR, giving the capacitor its initial condition.

```
s\tr("r2,0,1,2:r5,1,o,5:c,1,o,1/20,10:o,0,1,o”):{vc,ic,vo}
```

$\{ 10\, e^{-4t} , -2\, e^{-4t} , -10\, e^{-4t} \}$

# Bo2's Drill Exercise 5.5 (Op Amp)

Determine $v_C(t)$, $i_C(t)$ and $v_o(t)$ for all t.




**Solution.** For **t < 0s**, simulate in DC. This gives us the initial condition.

```
s\dc("e,2,0,4:o,2,1,o:c,o,1,1/20:r5,o,1,5:r2,1,0,2"):{vc,ic,vo}
```

**{ 10 , 0 , 14 }**

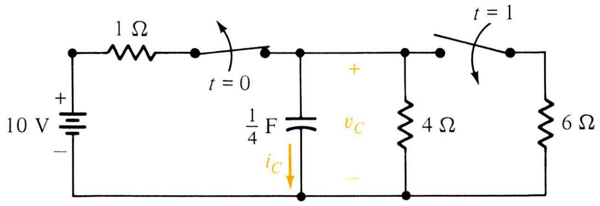
For **t ≥ 0s**, simulate in TR, giving the capacitor its initial condition.

```
s\tr("o,0,1,o:c,o,1,1/20,10:r5,o,1,5:r2,1,0,2”):{vc,ic,vo}
```

$\{ 10\, e^{-4t},\ -2\, e^{-4t},\ 10\, e^{-4t} \}$

## Bo2's Example 5.6

In the circuit shown below, there are two switches: one that opens at time t=0 and one that closes at time t=1 second. Determine $v_C(t)$ and $i_C(t)$ for all t.




As we mentioned before, every time a switch moves marks the end of one interval and the beginning of another interval for Symbulator.

For the first interval, which corresponds to t<0, we simulate in DC.

```
s\dc("e,2,0,10:r1,2,1,1:c,1,0,1/4:r4,1,0,4"):{vc,ic}
```

**{ 8 , 0 }**

The voltage in the capacitor at the end of this interval will serve as the initial condition of the capacitor for the next interval. The second interval runs between 0 and 1 second, e.g. 0 < t ≤ 1 second. We simulate in TR.

```
s\tr("c,1,0,1/4,8:r4,1,0,4”):{vc,ic}
```

$$\{ 8\, e^{-t} , -2\, e^{-t} \}$$

The voltage in the capacitor at the end of this second interval will serve as initial condition for the third interval. We can use its exact value, e.g. `8*e^-1`, but the textbook prefers using its approximate value, e.g. `2.943`.

The third interval corresponds to t > 1 second. We simulate in TR. To make things easier for Symbulator, we replace the resistors by their equivalent.

```
s\tr("c,1,0,1/4,2.943:re,1,0,[4,6]")
```

Ask for `{vc,ic}`. The expressions we get are equivalent to:

$$2.943\, e^{\frac{-5(t-1)}{3}} \qquad -1.226\, e^{\frac{-5(t-1)}{3}}$$

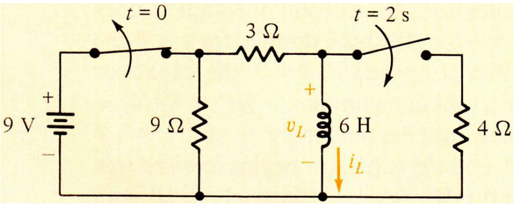
These are the right answers, as can be seen by checking the book's answers.

$$v_C(t) = \begin{cases} 8 \text{ V} & \text{for} \quad t < 0 \\ 8e^{-t} \text{ V} & \text{for} \quad 0 \le t < 1 \text{ s} \\ 2.943e^{-5(t-1)/3} \text{ V} & \text{for} \quad t \ge 1 \text{ s} \end{cases}$$

$$i_C(t) = \begin{cases} 0 \text{ A} & \text{for} \quad t < 0 \\ -2e^{-t} \text{ A} & \text{for} \quad 0 \le t < 1 \text{ s} \\ -1.226e^{-5(t-1)/3} \text{ A} & \text{for} \quad t \ge 1 \text{ s} \end{cases}$$

## Bo2's Drill Exercise 5.6

Let's see another example where a TR simulation is done for an interval that starts at a time other than t=0. Determine $i_L(t)$ and $v_L(t)$ for all t.




For the first interval, which corresponds to t<0, we simulate in DC.

```
s\dc("e,1,0,9:r9,1,0,9:r3,1,2,3:l,2,0,6"):{il,vl}
```

**{ 3 , 0 }**

The 3A current in the inductor is its initial condition for the second interval, which goes from 0 to 2 second, e.g. 0 < t ≤ 2 seconds. We simulate in TR.

```
s\tr("r,1,0,9+3:l,1,0,6,3"):{il,vl}
```

$$\{ 3\,e^{-2t}\ ,\ -36\,e^{-2t} \}$$

The current in the capacitor at the end of this second interval will serve as initial condition for the third interval. Find its approximate value thus:

```
iL|t=2.
```

**.055**

The third interval corresponds to t > 2 second. We simulate in TR. To make things easier for Symbulator, we replace the resistors by their equivalent.

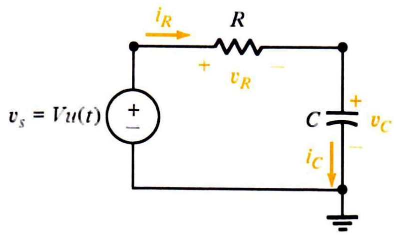
```
s\tr("r,1,0,[9+3,4]:l,1,0,6,.055"):{il,vl}
```

$$.055\,e^{\left(\frac{-(t-2)}{2}\right)} \qquad -.165\,e^{\left(\frac{-(t-2)}{2}\right)}$$

These are the kind of expressions your book or professor are looking for.

# Bo2's Figure 5.19

In the following circuit, assume that all the initial conditions are zero. The source is a step function ***u(t)*** with value **V** volts. Find $i_R$, $v_R$, $i_C$ and $v_C$.




As discussed before, a step source with symbolic value requires that we use the u(t) nomenclature to describe it. Since the value is V volts, we describe the source as V*u(t).

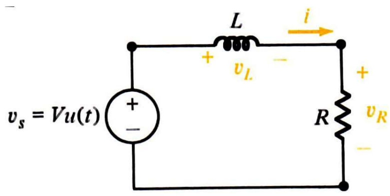
```
s\tr("e,1,0,V*u(t):r,1,2,r:c,2,0,c,0”):{vc,vr,ic}
```

There is no need to ask for $i_R$, since $i_R$=$i_C$. We get the following expressions:

$$\{\, v - v\, e^{\left(\frac{-t}{c\, r}\right)}\ ,\ v\, e^{\left(\frac{-t}{c\, r}\right)}\ ,\ \frac{v}{r} e^{\left(\frac{-t}{c\, r}\right)} \,\}$$

# Bo2's Figure 5.24

In the following circuit, assume that all the initial conditions are zero. The source is a step function ***u(t)*** with value **V** volts. Find $i_R$, $v_R$, $i_L$ and $v_L$.



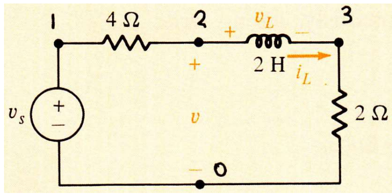

```
s\tr("e,1,0,V*u(t):l,1,2,l,0:r,2,0,r”):{vl,vr,il}
```

There is no need to ask for $i_R$, since in a series circuit it will be identical to $i_L$.

$$\{\, v\, e^{\frac{-r\, t}{l}}\ ,\ v\,(1 - e^{\frac{-r\, t}{l}})\ ,\ \frac{v}{r}(1 - e^{\frac{-r\, t}{l}}) \,\}$$

## Bo2's Drill Exercise 5.7

For the circuit below, with a step source of 12 volts, find $i_L$, $v_L$, $i_L$ and v.



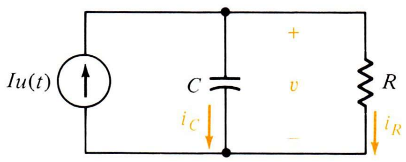

As discussed before, a step source with numerical value can be described without the u(t) nomenclature. Thus, we describe the source as 12.

s\tr("e,1,0,12:r4,1,2,4:l,2,3,2,0:r2,3,0,2”):{il,vl,v2}

$$\{\, 2 - 2e^{-3t} \,,\, 12\, e^{-3t} \,,\, 8\, e^{-3t} + 4 \,\}$$

## Bo2's Figure 5.26

In the following circuit, assume that all the initial conditions are zero. The source is a step function ***u(t)*** with value **I** amperes. Find $i_C$, $i_R$, and v.



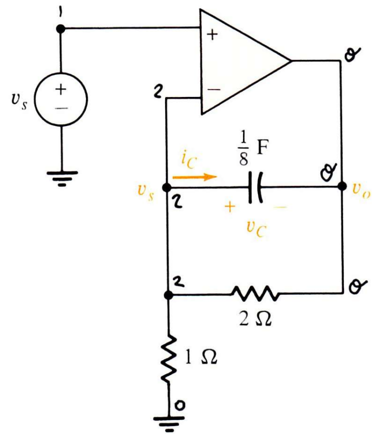

As discussed before, a step source with symbolic value requires that we use the u(t) nomenclature to describe it. Since the value is I amperes, we describe the source as I*u(t). Thus:

s\tr("j,0,1,i*u(t):c,1,0,c,0:r,1,0,r”):{v1,ir,ic}

$$\{\, \left(i - i\, e^{-\frac{t}{c\,r}}\right) * r \,,\, i - i\, e^{-\frac{t}{c\,r}} \,,\, i\, e^{-\frac{t}{c\,r}} \,\}$$

## Bo2's Example 5.7 (Op Amp)

Suppose that $v_s(t)=u(t)$. Find $v_C$, $i_C$, and $v_o$.



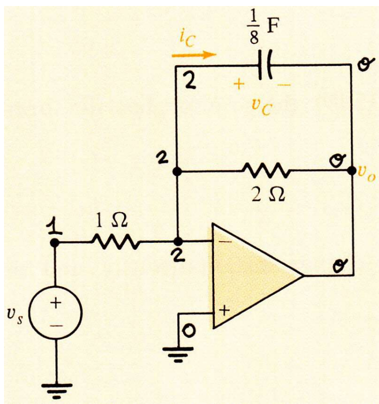

This source is a step source with a value of 1 volt. Since its value is numerical, the u(t) nomenclature is not needed. We describe the source plainly as 1.

```
s\tr("e,1,0,1:o,1,2,o:c,2,o,1/8,0:r2,2,o,2:r1,2,0,1"):{vc,ic,vo}
```

$$\{\, 2\,e^{-4t} - 2 \,,\, -e^{-4t} \,,\, 3 - 2e^{-4t} \,\}$$

## Bo2's Drill Exercise 5.8 (Op Amp)

Suppose that $v_s(t)=u(t)$. Find $v_C$, $i_C$, and $v_o$.



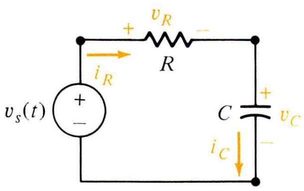

Again, following the same logic as above, we describe the source plainly as 1.

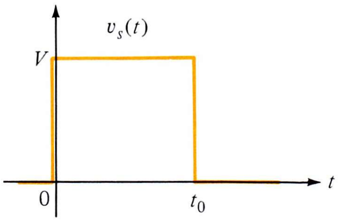
```
s\tr("e,1,0,1:r1,1,2,1:r2,2,o,2:c,2,o,1/8,0:o,0,2,o"):{vc,ic,vo}
```

$$\{\, 2 - 2\,e^{-4t} \,,\, e^{-4t} \,,\, 2\,e^{-4t} - 2 \,\}$$

## Bo2's p249 F5.28

For the circuit left, the source is $v_s(t) = V\,u(t) - V\,u(t-t_0)$. This is shown in the plot to the right. Given this, find the voltage drop in the capacitor for all $t$.




Because the source has two steps, this problem has to be analyzed in two intervals. For the first interval, the initial conditions are zero and the source value is V*u(t). Our circuit description for the first interval is given below.

s\tr("e,1,0,V*u(t):r,1,2,r:c,2,0,c,0”):vc

The expression for the capacitor's voltage drop in the first interval is:

$$v - v\,e^{\left(\frac{-t}{c\,r}\right)}$$

The second interval starts at $t_0$. In it, the initial condition of the capacitor is given by the value of the expression above when $t=t_0$. Here is how we find it:

vc|t=to

$$v - v\,e^{\left(\frac{-to}{c\,r}\right)}$$

The circuit description for the second interval uses the expression above as the initial condition of the capacitor. In this second interval, the source has a value of 0 volts, which is another way to say that it becomes a short, so there is no need to include it in the circuit description.

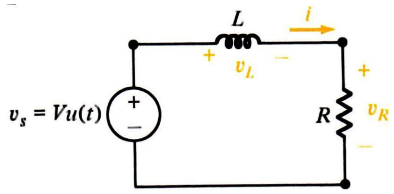
s\tr("r,0,2,r:c,2,0,c,v-e^(-to/(c*r))*v"):vc

$$\left(v - v\,e^{\left(\frac{-to}{c\,r}\right)}\right)e^{\left(\frac{-t+to}{c\,r}\right)}$$

This is the expression for the capacitor's voltage drop in the second interval.

## Bo2's Drill Exercise 5.9

Find the current through the inductor if the source is $v_s(t) = V\,u(t) - V\,u(t-t_o)$.




Because the source has two steps, this problem has to be analyzed in two intervals. For the first interval, the initial conditions are zero and the source value is V*u(t). Our circuit description for the first interval is given below.

s\tr("e,1,0,V*u(t):l,1,2,l,0:r,2,0,r"):il

The expression for the inductor's current in the first interval is:

$$\frac{v}{r}\left(1-e^{\left(\frac{-r\,t}{l}\right)}\right)$$

The second interval starts at $t_0$. In it, the initial condition of the inductor is given by the value of the expression above when $t=t_0$. Here is how we find it:

il|t=to

$$\frac{v}{r}\left(1-e^{\left(\frac{-to\,r}{l}\right)}\right)$$

The circuit description for the second interval uses the expression above as the initial condition of the inductor. In this second interval, the source becomes a short, so there is no need to include it in the circuit description.

s\tr("l,0,2,l,v/r-e^(-to*r/l)*v/r:r,2,0,r"):il

$$\frac{v}{r}\left(1-e^{\left(\frac{-to\,r}{l}\right)}\right)e^{\left(\frac{-r\,t}{l}\right)}$$

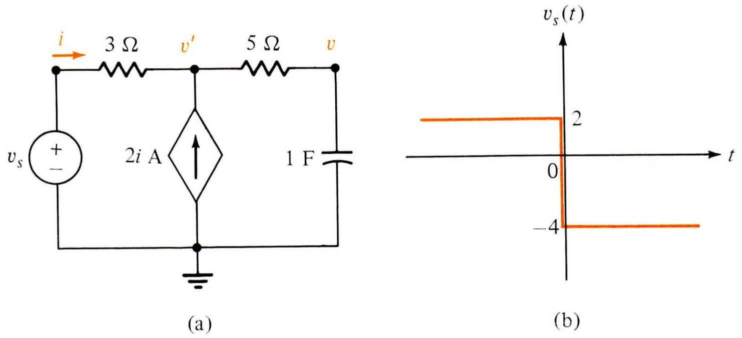
To put this in terms of the same t as the first interval, we replace ***t*** with ***t-t_o***:

$$\frac{v}{r}\left(1-e^{\left(\frac{-to\,r}{l}\right)}\right)e^{\left(\frac{-r\,(t-to)}{l}\right)}$$

This is the expression for the inductor's current in the second interval.

## Bo2's Figure 5.36

For the circuit shown in (a), find the voltage drop in the capacitor, given that the voltage source is as shown in (b).




This problem requires two intervals. In the first interval, a DC analysis of the circuit with a source of 2 V gives us the voltage in the capacitor.

```
s\dc("e,1,0,2:r3,1,2,3:r5,2,3,5:c,3,0,1:j,0,2,2ir3"):vc
```

$$\mathbf{2}$$

This is the initial condition for the capacitor in the second interval, which we analyze using TR and a source value of -4 V.

```
s\tr("e,1,0,-4:r3,1,2,3:r5,2,3,5:c,3,0,1,2:j,0,2,2ir3”):vc
```

$$6e^{\left(\frac{-t}{6}\right)} - 4$$

This is the expression for the voltage drop in the capacitor after t=0.

# The **only** tool

## Bo2's Drill Exercise 5.11

For the circuit of Bo2's Example 5.7, change the value of the capacitor to ¼ F. Find $v_C(t)$ for the case that the source is $v_S(t)$=1 V for t<0 and 3V for t≥0.

We let Symbulator know that we are only interested in one variable: vc.

```
s\only("vc")
```

Then we run the simulation for the first interval, just as we did before, but with the new capacitor and source values.

```
s\dc("e,1,0,1:o,1,2,o:c,2,o,1/4:r2,2,o,2:r1,2,0,1"):vc
```

**-2**

This is the initial condition for the next interval. Before we run the simulation for the second interval, we let Symbulator know, again, that we are only interested in one variable: vc.

```
s\only("vc")
```

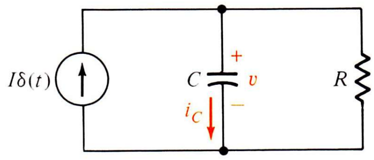
Then we run the simulation for the second interval, just as we did before, but with the new capacitor and source values.

```
s\tr("e,1,0,3:o,1,2,o:c,2,o,1/4,-2:r2,2,o,2:r1,2,0,1”):vc
```

$$\boldsymbol{4e^{(-2t)} - 6}$$

## Bo2's Figure 5.38 (Impulse)

Find the voltage drop and current in the capacitor, given a source of I δ(t) A, where δ(t) is the impulse function and I is a constant.




As we mentioned before, impulse sources must be described using the δ(t) nomenclature. To save some typing time, you find it in the menu.

s\tr("j,0,1,i*δ(t):c,1,0,c,0:r,1,0,r”)

Once the simulation completes, we ask for the variables of interest:

vc

$$\left(\frac{i}{c}\right) e^{\left(\frac{-t}{c\,r}\right)}$$

ic

$$\left(\frac{-i}{c\,r}\right) e^{\left(\frac{-t}{c\,r}\right)}$$

## Bo2's Drill Exercise 5.13 (Impulse)

For the circuit of Bo2's Example 5.7, find $v_C(t)$, $i_C(t)$ and $v_o(t)$ due to an impulse voltage source of $v_S(t)$= δ(t) (that is to say, a 1V impulse in t=0.)

Nothing new here. Again we use the δ(t) nomenclature for the source.

s\tr("e,1,0,δ(t):o,1,2,o:c,2,o,1/8,0:r2,2,o,2:r1,2,0,1”):{vc,ic,vo}

$$\{\, -8\, e^{-4t} \,,\; 4\, e^{-4t} \,,\; 8\, e^{-4t} \,\}$$

## Bo2's Figure 5.39 (Ramp)

Find iL(t) and vL(t) for a circuit that has in parallel a resistor R, an inductor L and a current source with a ramp value I r(t). Initial conditions are zero.

A ramp value I r(t) means a value of I t. We describe this source simply as `i*t`.

```
s\tr("j,0,1,i*t:r,1,0,r:l,1,0,l,0”):{il,vl}
```

$$\{\frac{i\,l}{r}\left(e^{\left(\frac{-r\,t}{l}\right)}-1\right)+i\,t\,,\,i\,l\left(1-e^{\left(\frac{-r\,t}{l}\right)}\right)\}$$

## Bo2's Drill Exercise 5.14 (Ramp)

For the circuit of Bo2's Example 5.7, find $v_C(t)$ and $i_C(t)$ due to a ramp input voltage of $v_S(t)$= r(t).

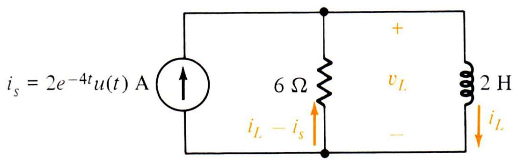
Nothing new here. The command below should be clear to you by now.

```
s\tr("e,1,0,t:o,1,2,o:c,2,o,1/8,0:r2,2,o,2:r1,2,0,1”):{vc,ic}
```

$$\{\frac{1}{2}(1-e^{-4t})-2t\,,\,\frac{1}{4}(e^{-4t}-1)\}$$

## Bo2's Example 5.12 (Exponential)

Assume initial conditions zero. Find $i_L(t)$ and $v_L(t)$, given that $i_S(t) = 2\,e^{-4t}$ A.




Describe the source as given to you by the problem, using an exponential.

```
s\tr("j,0,1,2e^(-4t):r,0,1,6:l,1,0,2,0”):{il,vl}
```

$$\{6\,e^{-3t}-6\,e^{-4t}\,,\,48\,e^{-4t}-36\,e^{-3t}\}$$

## Bo2's p265 (Exponential)

Repeat Bo2's Example 5.12 if the current source is now $i_S(t) = 2\,e^{-3t}$ A.

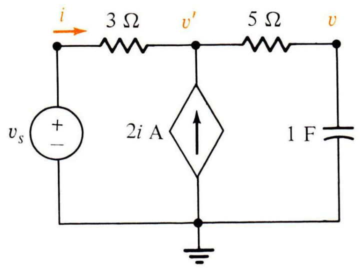
All we do is change one digit in the circuit description: replace 4 with 3.

s\tr("j,0,1,2e^(-3t):r,0,1,6:l,1,0,2,0"):{il,vl}

$$\{\, 6\, t\, e^{-3t}\,,\ (12 - 36\, t)\, e^{-3t} \,\}$$

## Bo2's Example 5.13 (Exponential)

Find the voltage drop in the capacitor and the current in the 3 Ω resistor, if the source value is $v_S(t)=18e^{-t/2}$.




s\tr("e,1,0,18e^(-t/2):r3,1,2,3:r5,2,3,5:c,3,0,1,0:j,0,2,2ir3"):{vc,ir3}

$$\{\, 9\, e^{\left(\frac{-t}{6}\right)} - 9\, e^{\left(\frac{-t}{2}\right)}\,,\ \frac{3}{2}\, e^{\left(\frac{-t}{2}\right)} - \frac{1}{2} e^{\left(\frac{-t}{6}\right)} \,\}$$

## Bo2's Drill Exercise 5.16 (Exponential)

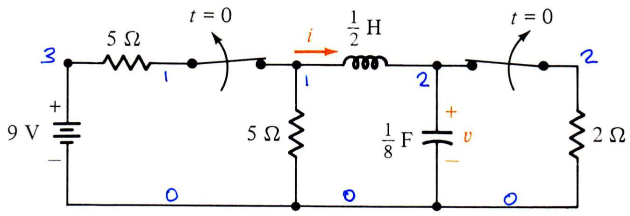
For the circuit of Bo2's Example 5.7, find $v_C(t)$ and $i_C(t)$ for the case when the source's value is $v_S(t)= 2\,e^{-4t}$. As before, assume initial conditions are zero.

s\tr("e,1,0,2e^(-4t):o,1,2,o:c,2,o,1/8,0:r2,2,o,2:r1,2,0,1"):{vc,ic}

$$\{\, -16\, t\, e^{-4\,t}\,,\ (8\, t - 2)\, e^{-4t} \,\}$$

## Bo2's Example 6.1

Find the current in the inductor i(t) and the voltage drop in the capacitor v(t).




Since we have switches changing in time t=0, this is a two interval problem. The first interval is for t<0. We can analyze the first interval using DC.

s\dc("e,3,0,9:r2,3,1,5:r1,1,0,5:l,1,2,1/2:c,2,0,1/8:r3,2,0,2"):{vc,il}

**{2,1}**

These are the initial conditions for the second interval. The second is for t≥0. We can analyze the second interval using TR. We only care about vc and il.

s\only("vc,il"):s\tr("r1,1,0,5:l,1,2,1/2,1:c,2,0,1/8,2"):{vc,il}

$$\{\, 4\,e^{-2t} - 2\,e^{-8t} \,,\, 2\,e^{-8t} - e^{-2t} \,\}$$

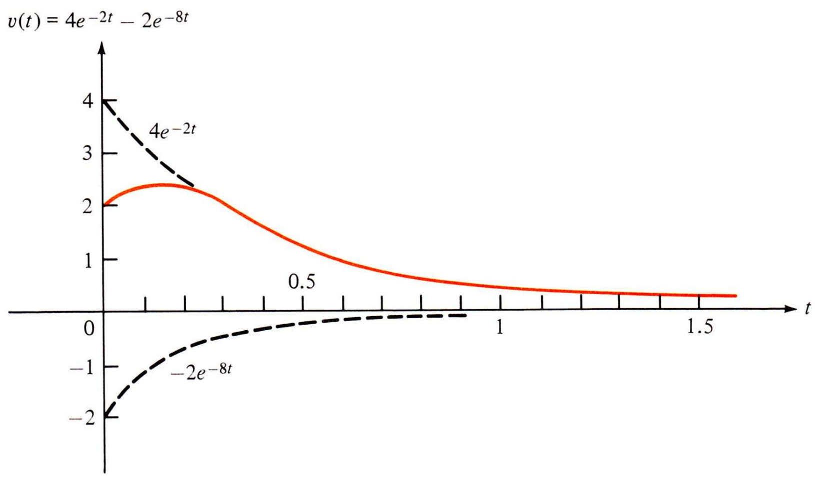
Notice that the use of the only tool saves approximately 10 seconds here.

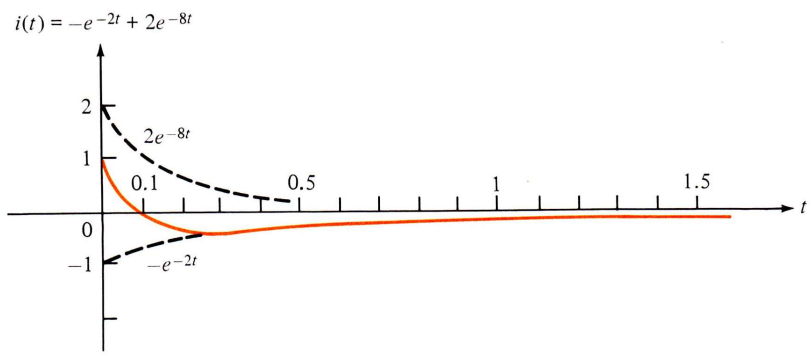
# The plot tool

Imagine, for example, that in Bo2's Example 6.1 we are asked to plot i(t) and v(t) for times between 0 s and 1.5 s. The first step is to run the plot tool:

s\plot()

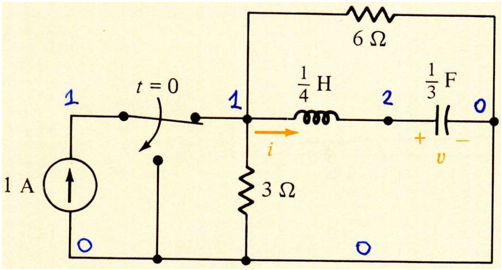
A window opens, asking you to enter three things: first, a function of time; second, a minimal time; and third, a maximal time. Let's plot $v_C(t)$ first. Unless you have deleted the value of the variable **vc**, you should have the answer to the simulation above stored in it. So enter vc as the function, 0 as the minimal time and 1.5 as the maximal time. Press Enter, and wait a little. The graph of the voltage drop in the capacitor between time 0 and 1.5 seconds should appear in the screen. Compare it to the graph given by the textbook, shown below. Repeat this procedure giving ic as the function. Compare the resulting graph to that given by the textbook, shown below.



## Bo2's Drill Exercise 6.1

For the circuit shown below, find v(t) and i(t) for t≥0.




Nothing new here. This is a two interval problem, as others seen before. The first interval, for t<0, is analyzed in DC to find the initial conditions.

```
s\dc("j,0,1,1:r3,1,0,3:r6,1,0,6:l,1,2,1/4:c,2,0,1/3"):{vc,il}
```

**{2,0}**

These conditions are used in the TR analysis of the second interval, for t≥0.

```
s\only("vc,il"):s\tr("r3,1,0,3:r6,1,0,6:l,1,2,1/4,0:c,2,0,1/3,2”)
```

vc

$$3e^{-2t} - e^{-6t}$$

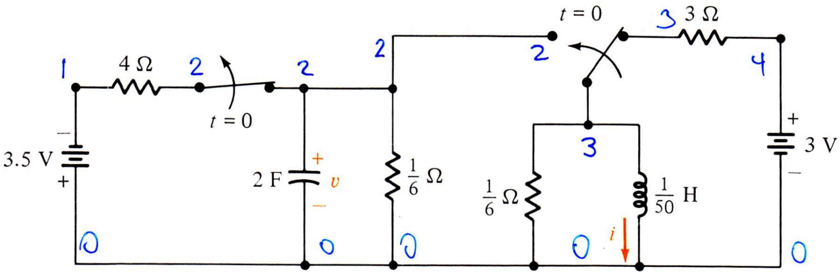
il

$$2e^{-6t} - 2e^{-2t}$$

Using only saves about 13 seconds, or about a third of the calculation time, because only about 8 seconds are used finding the inverse Laplace of the selected answers, instead of 21 seconds finding it for all the answers.

## Bo2's Example 6.2

Find i(t) and v(t) for the circuit shown below.




I decided to simulate this using only fractions, not decimals. So I convert the 3.5V value to its exact fractional equivalent, thus:

```
exact(3.5)
```

**7/2**

Because of the switches, this is a two interval problem. The first interval is to be analyzed using DC. Notice that for t<0 the circuit is such that we basically have two separate circuits. We simulate each one of them separately:

```
s\dc("e1,0,1,7/2:r1,1,2,4:c,2,0,2:r2,2,0,1/6"):vc
```

**-7/50**

```
s\dc("r3,3,0,1/6:l,3,0,1/50:r4,3,4,3:e2,4,0,3"):il
```

**1**

The second interval, for t≥0, is analyzed using TR. I could have described the circuit again from scratch. But out of laziness I preferred to copy/paste the descriptions from the DC simulations. To avoid renaming the nodes, I simulated the right switch as a short circuit between nodes 2 and 3.

```
s\only("vc,il"):s\tr("c,1,0,2,-7/50:r,1,0,[1/6,1/6]:l,1,0,1/50,1”)
```

```
vc
```

$$\frac{-7}{50}e^{-3t}\cos(4t)-\frac{1}{50}e^{-3t}\sin(4t)$$

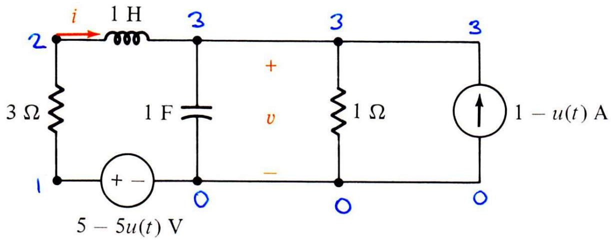
```
il
```

$$e^{-3t}\cos(4t)-e^{-3t}\sin(4t)$$

The TR simulation took 30 s in my calculator. These are the right answers.

## Bo2's Example 6.3

Find i(t) and v(t) for the circuit shown below.




Since the sources change value, this is a two interval problem. For t<0, in DC:

```
s\dc("e,1,0,5:r3,1,2,3:l,2,3,1:c,3,0,1:r1,3,0,1:j,0,3,1"):{vc,il}
```

**{2,1}**

Using these as initial conditions, now we use TR for t≥0:

```
s\only("vc,il"):s\tr("r3,0,2,3:l,2,3,1,1:c,3,0,1,2:r1,3,0,1”):{vc,il}
```

$$\{\ (3t+2)e^{-2t}\ ,\ (1-3t)e^{-2t}\ \}$$

# Is the textbook wrong?

Anybody that has even written a textbook, or read one for that matter, can attest to the fact that errors in the problems or the answers come with the territory. (As a matter of fact, I am sure this book I am writing right now has many errors.) We are all only human. Errors in an engineering textbook, however, can be *very* frustrating to an engineering student. When one is learning from a textbook, and the textbook has errors in the problems or answers, confusion and hair-pulling will likely ensue. If you think you have found an error in your circuit analysis textbook, Symbulator can help you bring some sanity to the situation to confirm whether your guess is right.

Below is an example of using Symbulator to confirm an error in a textbook.

## Bo2's Drill Exercise 6.4

DRILL EXERCISE 6.4

For the *RLC* circuit shown in Fig. DE6.4, suppose that $v_s(t) = 2 - 2u(t)$ V. Find $i(t)$ and $v(t)$ for $t \geq 0$.

Answer: $8te^{-4t} + 2e^{-4t}$ A; $-16te^{-4t}$ V

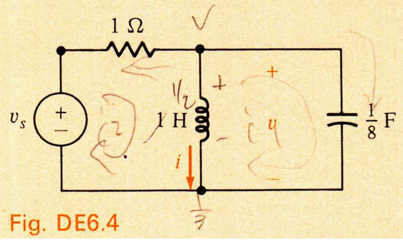



Fig. DE6.4

My copy of Bo2 is second-hand. I noticed this problem had scribbles on it. The previous owner, evidently, struggled with this problem and concluded that the value of the inductor had to be ½ H, not 1 H as the schematic says. Symbulator can be used to show that he is right, and the book is wrong. Here is how we do it. First, we can solve the circuit as shown, with a 1 H inductor.

```
s\dc("e,2,0,2:r,1,2,1:l,1,0,1:c,1,0,1/8"):{il,vc}
```

**{2,0}**

```
s\only("il,vc"):s\tr("r,1,0,1:l,1,0,1,2:c,1,0,1/8,0”):{il,vc}
```

The answer we get doesn't match the answer provided by the book. Now let's repeat the TR simulation using the ½ H value for the inductor:

```
s\only("il,vc"):s\tr("r,1,0,1:l,1,0,1/2,2:c,1,0,1/8,0”):{il,vc}
```

Now the answer we get matches the answer provided by the book. An expert user of Symbulator can use the `fd` gate and the `t2s` tool, which we have not discussed yet here, to find the value of the inductor given the book's answer. Below is how I found the ½ H value myself, reverse-engineering the answer:

```
s\t2s(-16*t*e^(-4*t)):
```

**-16/(s+4)^2**

```
s\t2s((8*t+2)*e^(-4*t))
```

**2*(s+8)/(s+4)^2**

```
s\fd("r,1,0,1:l,1,0,l,2:c,1,0,1/8,0"):vc
```

**-16*l/(l*s^2+8*l*s+8)**

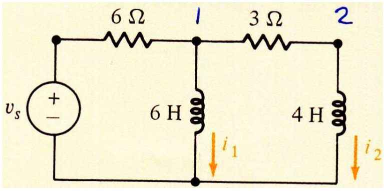
```
solve(-16/(s+4)^2=-16*l/(l*s^2+8*l*s+8),l)
```

**l=1/2**

The `fd` gate and `t2s` tool are used for frequency domain analysis, and will be discussed separately in a subsequent part of Symbulator's documentation.

## Bo2's Drill Exercise 6.5

In the circuit below, suppose that $v_S(t)= 12 – 12\ u(t)$ V. Find $i_2(t)$ for $t≥0$.




s\dc("e,3,0,12:r6,3,1,6:l1,1,0,6:r3,1,2,3:l2,2,0,4"):{il1,il2}

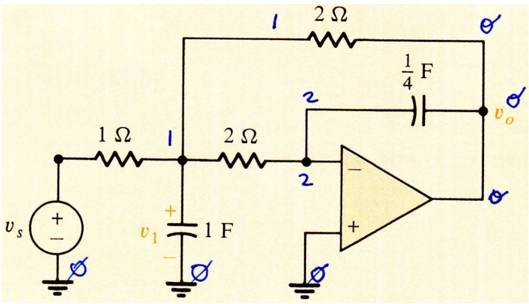
**{2,0}**

s\only("il2"):s\tr("r6,0,1,6:l1,1,0,6,2:r3,1,2,3:l2,2,0,4,0”):il2

$$\frac{12}{11}\left(e^{-3t} - e^{-t/4}\right)$$

## Bo2's Drill Exercise 6.6 (Op Amp)

In the circuit below, suppose that $v_S(t)= 2 – 2\ u(t)$ V. Find $v_o(t)$ for $t≥0$.




Simulate the first interval in DC to find the initial conditions of the capacitors.

s\dc("e,3,0,2:r1,3,1,1:r2,1,2,2:r3,1,o,2:ca,1,0,1:
cb,2,o,1/4:o,0,2,o"):{vca,vcb}

**{0,4}**

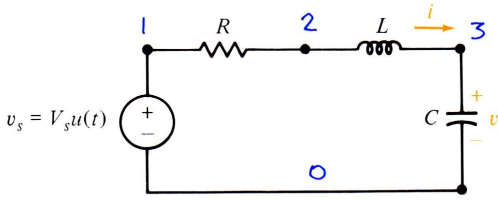
Then simulate the second interval in TR, to find the voltage in node *o*.

s\only("vo"):s\tr("r1,0,1,1:r2,1,2,2:r3,1,o,2:
ca,1,0,1,0:cb,2,o,1/4,4:o,0,2,o”):vo

$$(-4t - 4)e^{-t}$$

## Bo2's Example 6.5 (Plot)

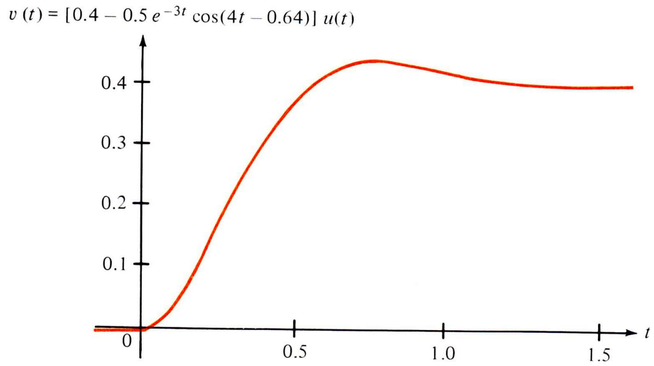
In the circuit below, use $v_S$=2/5V, R=12Ω, L=2H and C=1/50F. Find v(t) and i(t), and plot them for time 0<t<1.5 seconds.

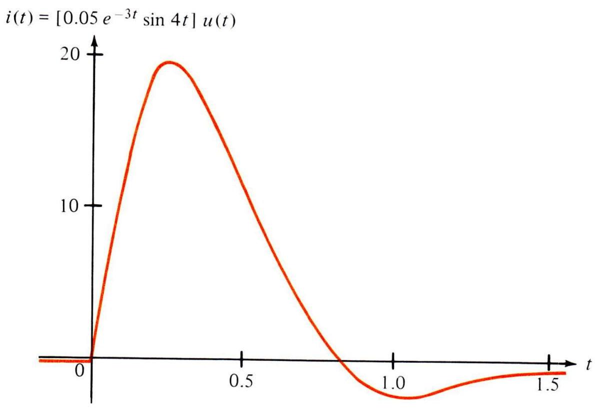



s\only("vc,il"):s\tr("e,1,0,2/5:r,1,2,12:l,2,3,2,0:c,3,0,1/50,0”)

| vc | il |
|---|---|
| $-\frac{2}{5}e^{-3t}cos(4t) - \frac{3}{10}e^{-3t}sin(4t) + \frac{2}{5}$ | $\frac{1}{20}e^{-3t}sin(4t)$ |

These are the right answers. To plot them, run the plot tool, thus: s\plot(). Once the plot window opens, enter vc as function, 0 as minimal time and 1.5 as maximal time. Compare to the graph from the book, shown below. Repeat the plot for il, and compare to the graph from the book below.



## Bo2's Drill Exercise 6.7

For the circuit in Bo2's Example 6.5, use $v_S$=3V, R=5Ω, L=1/2H and C=1/8F. Find v(t) and i(t).

```
s\only("vc,il"):s\tr("e,1,0,3:r,1,2,5:l,2,3,1/2,0:c,3,0,1/8,0”):{vc,il}
```

$$\{ -4e^{-2t} + e^{-8t} + 3 \, , \, e^{-2t} - e^{-8t} \}$$

## Bo2's Drill Exercise 6.8 (Ramp)

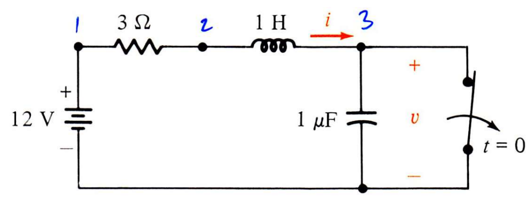
For the circuit in Bo2's Drill Exercise 6.7, find the voltage drop in the capacitor v(t) if the source has a value $v_S(t)$ = 3 r(t).

```
s\only("vc"):s\tr("e,1,0,3t:r,1,2,5:l,2,3,1/2,0:c,3,0,1/8,0”):vc
```

$$2e^{-2t} - \frac{e^{-8t}}{8} + 3t - \frac{15}{8}$$

## Bo2's Example 6.6 (Plot)

For the circuit below, plot the voltage drop in the inductor for 0 < t < 5ms.




First, find the initial conditions for t<0 by running a DC simulation.

```
s\dc("e,1,0,12:r,1,2,3:l,2,3,1:c,3,0,1’μ:s,3,0"):{il,vc}
```

**{4,0}**

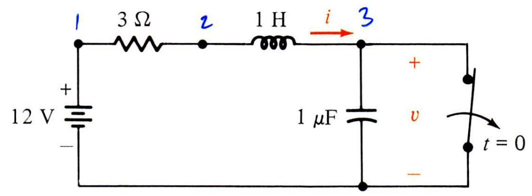
Then, find the vl for t≥0 by running a TR simulation.

```
s\only("vl"):s\tr("e,1,0,12.:r,1,2,3:l,2,3,1,4:c,3,0,1’μ,0”)
```

Finally, run s\plot() and enter vl as function, 0 as minimal time and 0.005 as maximal time. In the resulting plot you will see something outstanding: around t = 1.57 ms, the voltage drop across the inductor is 3991 volts!

## Bo2's Drill Exercise 6.9

For the circuit below, R=1Ω, L=2H, C=1/2F and $i_S(t)$=u(t). Find i(t) and v(t).

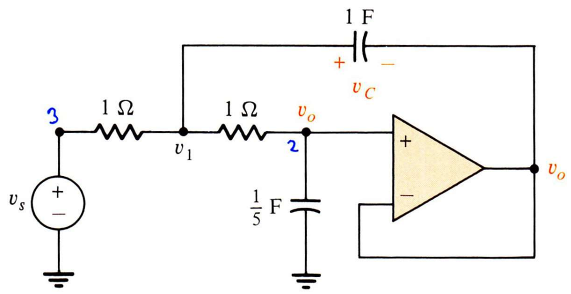



s\only("il,vc"):s\tr("j,0,1,1:r,1,0,1:l,1,0,2,0:c,1,0,1/2,0”):{il,vc}

$$\{\ (-t-1)e^{-t}+1\ ,\ 2te^{-t}\ \}$$

## Bo2's Figure 6.23 (Op Amp)

For the circuit below, find $v_o(t)$ if $v_S(t)$ = u(t).




s\only("vo"):s\tr("e,3,0,1:r1,3,1,1:r2,1,2,1:
ca,2,0,1/5,0:cb,1,o,1,0:o,2,o,o”):vo

$$1-e^{-t}\cos(2t)-\frac{1}{2}e^{-t}\sin(2t)$$

## Bo2's Drill Exercise 6.10 (Op Amp)

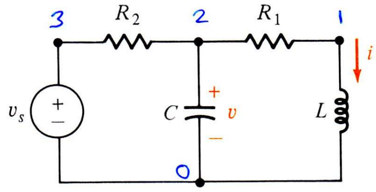
In the circuit above, change capacitor's value from 1/5 F to 25/16 F. Find $v_o(t)$ if $v_S(t)$ = 3 u(t).

s\only("vo"):s\tr("e,3,0,3:r1,3,1,1:r2,1,2,1:
ca,2,0,25/16,0:cb,1,o,1,0:o,2,o,o”):vo

$$-4e^{\frac{-2t}{5}}+e^{\frac{-8t}{5}}+3$$

## Bo2's Drill Exercise 6.11 (p307)

For the circuit below, suppose that $R_1 = R_2 = 1\Omega$, $L = 1H$, $C = 1F$ and $v_S(t) = 2e^{-2t}\, u(t)$. Find i(t) if all initial conditions are zero.

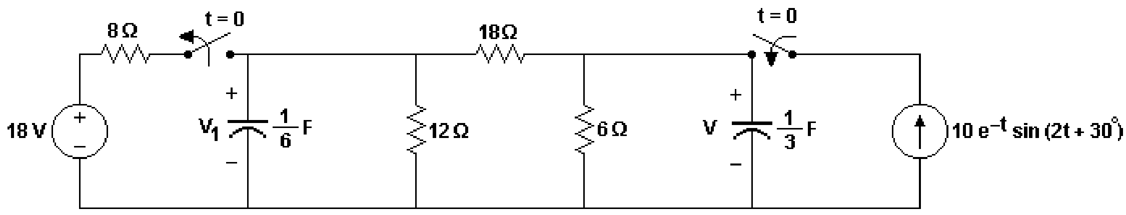



s\only("il"):s\tr("e,3,0,2e^(-2t):r2,3,2,1:r1,2,1,1:c,2,0,1,0:l,1,0,1,0”):il

$$-(e^t \cos(t) - e^t \sin(t) - 1)\, e^{-2t}$$

## A more complex problem

The circuit below comes from the circuits analysis class of Professor Eliane Boulet de Cabrera, from Universidad Tecnologica de Panama. Find v1 and v.




Because of the switches, this is a two interval problem. The first is run in DC.

```
s\dc("e,3,0,18:r8,3,1,8:ca,1,0,1/6:r12,1,0,12:
r18,1,2,18:r6,2,0,6:cb,2,0,1/3"):{vca,vcb}
```

**{9,9/4}**

These serve as initial conditions for the second interval, which is run in TR. Since we only want v1 and v2, it pays off to let Symbulator know this:

```
s\only("v1,v2"):
```

```
s\tr("ca,1,0,1/6,9:r12,1,0,12:r18,1,2,18:r6,2,0,6:
cb,2,0,1/3,9/4:j,0,2,10e^(-t)sin(2t+30°)")
```

Be patient. This took 78 seconds in my calculator, including over half a minute just to find the inverse Laplace of the two desired answers.

v1

$$\left(5\frac{\sqrt{3}}{17}-\frac{20}{17}\right)e^{-t}\cos(2t)+\left(-20\frac{\sqrt{3}}{17}-\frac{5}{17}\right)e^{-t}\sin(2t)+\left(80\frac{\sqrt{3}}{17}+\frac{193}{34}\right)e^{-\frac{t}{2}}+\left(\frac{9}{2}-5\sqrt{3}\right)e^{-t}$$

v2

$$\left(-245\frac{\sqrt{3}}{34}-\frac{20}{17}\right)e^{-t}\cos(2t)+\left(\frac{245}{34}-20\frac{\sqrt{3}}{17}\right)e^{-t}\sin(2t)+\left(80\frac{\sqrt{3}}{17}+\frac{193}{34}\right)e^{-\frac{t}{2}}+\left(5\frac{\sqrt{3}}{2}-\frac{9}{4}\right)e^{-t}$$

These are the correct answers, which explains why Professor Boulet – who made the problem up - is a living legend among circuit students at UTP.

# Advanced use of *Expert* in TR

The **expert** tool can be very useful in transient analysis. Using it, however, requires some knowledge. Here's two things you need to know in order to use ex like a boss:

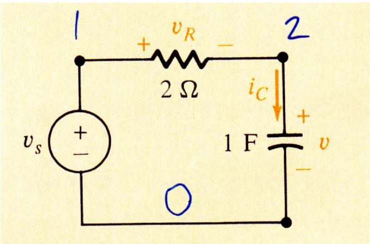
First, when Symbulator solves a problem using TR, it follows these general steps:

1. Generate a set of equations and unknowns for the system, in the frequency domain. Any time-dependent source is replaced with a dummy variable.
2. Solve these frequency domain equations.
3. Replace any dummy variable with the original source value, and convert the answers to the time domain.

Second, when Symbulator solves a problem using the expert tool, it freezes this process halfway between steps 1 and 2, so that you can tinkle with the equations and unknowns before they are solved. In the problems below, you will see that, when we tinkle with the equations, we have to do so in the frequency domain.

## Bo2's Drill Exercise 4.5 (Expert)

For this circuit, we know that the capacitor's initial condition is zero, that $v_s$ is an unknown step source (e.g. of the form A *u(t)*, where A is a constant value in volts) and that the voltage drop in the capacitor for t>0 is found to be 1-e^(-t/2). Find the voltage drop in the resistor, the current through the capacitor and the value of the source.




This problem is a match made in heaven for the **expert** tool, because we have one unknown value in the circuit (e.g. the value of the step source) and we have one known answer (e.g. the voltage drop in the capacitor.) So, the game plan here is to run this circuit through Symbulator's expert mode, add one new equation and one new unknown, and then solve. First, let's generate the new equation. A non-expert user would think that the new equation is 1-e^(-t/2)=vc. But you know better. You know that Symbulator solves the equations of TR problems in the frequency domain. So we have to convert this expression from the time domain to the frequency domain. Symbulator has a shortcut to invoke DiffEq's Laplace Transform to do so: s\t2s.

s\t2s(1-e^(-t/2))=vc

$$\frac{1}{s} - \frac{2}{2 * s + 1} = vc$$

We have our new equation. Copy this equation into the clipboard, since we will want to paste it in the Expert window. Now let's run the Expert simulation of the circuit. Let's define the value of the source as a*u(t), since we know it's a step source; the variable a will serve as the unknown value, for which we will solve in the Expert mode.

s\ex("e,1,0,a*u(t):r,1,2,2:c,2,0,1,0")

When prompted, select TR as the desired option. Then you will see the typical Expert prompt. In the equations field, paste the new equation, adding the word and first:

and 1/s-2/(2*s+1)=vc

In the unknown field, add the variable a, preceded by a comma:

,a

Let Symbulator solve the equations now. Once it is done, we want to do a quick sanity check. If the system solved correctly, we should see that Symbulator has as voltage drop in the capacitor the expression we already know from the problem statement:

vc

$$1 - e^{-\frac{t}{2}}$$

This is correct, so we proceed to ask for the answers we want:

ic

$$\frac{1}{2} e^{-\frac{t}{2}}$$

vr

$$e^{-\frac{t}{2}}$$

a

$$1$$

These are the correct answers. The value of the step source is 1 volt.

## Bo2's Drill Exercise 4.13 (Expert)

For the circuit shown in Fig. DE4.13, the voltage across the 1-Ω resistor is $v_R(t) = e^{-t}$ V for $t \geq 0$. Given that $v_C(0) = 0$ V, find the following functions for $t \geq 0$: (a) $i_R(t)$; (b) $v_C(t)$; and (c) $v_s(t)$.
Answer: (a) $e^{-t}$ A; (b) $1 - e^{-t}$ V; (c) 1 V

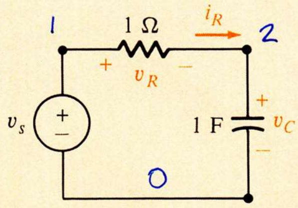


Fig. DE4.13

This problem is solved just like the one we already saw. First, find the new equation:

vr=s\t2s(e^(-t))

$$vr = \frac{1}{s+1}$$

Then run the Expert simulation. I decided to use vs as the step value of the source:

s\ex("e,1,0,vs*u(t):r,1,2,1:c,2,0,1,0"):

When prompted, select TR. In the equations field, add the new equation you found:

and vr=1/(s+1)

In the unknowns field, add the new variable:

,vs

Let Symbulator rip. Once it finishes solving, ask for the sanity check:

vr

$$e^{-t}$$

Looks good, so go ahead and ask for the answers:

ir

$$e^{-t}$$

vc

$$1 - e^{-t}$$

vs

$$1$$

Ain't that a beauty? Yes, ma'am! Atta boy, Symbulator! Atta boy!

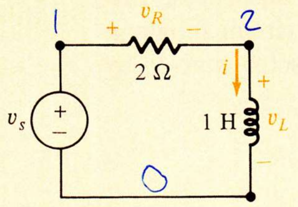
## Bo2's 164 (Expert)

For the circuit shown in Fig. DE4.2, the current through the inductor is

$$i(t) = \begin{cases} 0 & \text{for } -\infty < t < 0 \\ 1 - e^{-2t} & \text{for } 0 \le t < \infty \end{cases}$$

Find, for $t \ge 0$, (a) $v_L(t)$, (b) $v_R(t)$, (c) $v_s(t)$, and (d) the power absorbed by the inductor.

Answer: (a) $2e^{-2t}$ V; (b) $2(1 - e^{-2t})$ V; (c) 2 V; (d) $2(e^{-2t} - e^{-4t})$ W




This is the same thing as the previous two, so there is no need for commentary.

il=s\t2s(1-e^(-2*t))

$$il = \frac{1}{s} - \frac{1}{s+2}$$

s\ex("e,1,0,vs*u(t):r,1,2,2:l,2,0,1,0")

Choose TR, and add ,vs to the unknowns and the following to the equations

and il=1/s-1/(s+2)

And solve. When done, ask for the sanity check:

il

$$1 - e^{-2t}$$

Since that adds up, ask for the answers:

vl

$$2e^{-2t}$$

vr

$$2 - 2e^{-2t}$$

vs

$$2$$

vl*il

$$2e^{-4t}(e^{2t} - 1)$$

This is the end of the TR section of the book.